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# CHAPTER 5

## NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA

### Answers to Concepts Review and Critical Thinking Questions

1. Assuming conventional cash flows, a payback period less than the project's life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project's life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR. The discounted payback includes the effect of the relevant discount rate. If a project's discounted payback period is less than the project's life, it must be the case that NPV is positive.
2. Assuming conventional cash flows, if a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus, the payback period must be less than the project life. Since discounted payback is calculated at the same discount rate as is NPV, if NPV is positive, the discounted payback period must be less than the project's life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus, PI must be greater than 1. If NPV is positive for a certain discount rate  $R$ , then it will be zero for some larger discount rate  $R^*$ ; thus, the IRR must be greater than the required return.
3.
  - a. Payback period is simply the accounting break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that pay back before this cutoff, and reject projects that take longer to pay back. The worst problem associated with the payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for the payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.
  - b. The IRR is the discount rate that causes the NPV of a series of cash flows to be identically zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The acceptance and rejection criteria are:
    - If  $C_0 < 0$  and all future cash flows are positive, accept the project if the internal rate of return is greater than or equal to the discount rate.
    - If  $C_0 < 0$  and all future cash flows are positive, reject the project if the internal rate of return is less than the discount rate.
    - If  $C_0 > 0$  and all future cash flows are negative, accept the project if the internal rate of return is less than or equal to the discount rate.
    - If  $C_0 > 0$  and all future cash flows are negative, reject the project if the internal rate of return is greater than the discount rate.

IRR is the discount rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and it also may ambiguously rank some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.

- c. The profitability index is the present value of cash inflows relative to the project cost. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one. The profitability index can be expressed as:  $PI = (NPV + cost)/cost = 1 + (NPV/cost)$ . If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the “bang for the buck” of each particular project.
- d. NPV is simply the present value of a project’s cash flows, including the initial outlay. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV. NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and it can differentiate between projects of different scale and time horizon. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and thus not certain, but this is a problem shared by the other performance criteria as well. A project with  $NPV = \$2,500$  implies that the total shareholder wealth of the firm will increase by \$2,500 if the project is accepted.

4. For a project with future cash flows that are an annuity:

$$\text{Payback} = I / C$$

And the IRR is:

$$0 = -I + C / \text{IRR}$$

Solving the IRR equation for IRR, we get:

$$\text{IRR} = C / I$$

Notice this is just the reciprocal of the payback. So:

$$\text{IRR} = 1 / \text{PB}$$

For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR, and the IRR is approximately equal to the reciprocal of the payback period.

5. There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.

6. The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (discounted payback, NPV, IRR, and profitability index) are really only slightly more difficult in practice.
7. Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the “revenues” from not-for-profit ventures are not tangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!
8. The statement is false. If the cash flows of Project B occur early and the cash flows of Project A occur late, then for a low discount rate the NPV of A can exceed the NPV of B. Observe the following example.

	$C_0$	$C_1$	$C_2$	IRR	NPV @ 0%
Project A	-\$1,000,000	\$0	\$1,440,000	20%	\$440,000
Project B	-\$2,000,000	\$2,400,000	\$0	20%	400,000

However, in one particular case, the statement is true for equally risky projects. If the lives of the two projects are equal and the cash flows of Project B are twice the cash flows of Project A in every time period, the NPV of Project B will be twice the NPV of Project A.

9. Although the profitability index (PI) is higher for Project B than for Project A, Project A should be chosen because it has the greater NPV. Confusion arises because Project B requires a smaller investment than Project A. Since the denominator of the PI ratio is lower for Project B than for Project A, B can have a higher PI yet have a lower NPV. Only in the case of capital rationing could the company’s decision have been incorrect.
10. *a.* Project A would have a higher IRR since initial investment for Project A is less than that of Project B, if the cash flows for the two projects are identical.  
*b.* Yes, since both the cash flows as well as the initial investment are twice that of Project B.
11. Project B’s NPV would be more sensitive to changes in the discount rate. The reason is the time value of money. Cash flows that occur further out in the future are always more sensitive to changes in the interest rate. This sensitivity is similar to the interest rate risk of a bond.
12. The MIRR is calculated by finding the present value of all cash outflows, the future value of all cash inflows to the end of the project, and then calculating the IRR of the two cash flows. As a result, the cash flows have been discounted or compounded by one interest rate (the required return), and then the interest rate between the two remaining cash flows is calculated. As such, the MIRR is not a true interest rate. In contrast, consider the IRR. If you take the initial investment, and calculate the future value at the IRR, you can replicate the future cash flows of the project exactly.

13. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the required return, then calculate the NPV of this future value and the initial investment, you will get the same NPV. However, NPV says nothing about reinvestment of intermediate cash flows. The NPV is the present value of the project cash flows. What is actually done with those cash flows once they are generated is not relevant. Put differently, the value of a project depends on the cash flows generated by the project, not on the future value of those cash flows. The fact that the reinvestment “works” only if you use the required return as the reinvestment rate is also irrelevant simply because reinvestment is not relevant in the first place to the value of the project.

One caveat: Our discussion here assumes that the cash flows are truly available once they are generated, meaning that it is up to firm management to decide what to do with the cash flows. In certain cases, there may be a requirement that the cash flows be reinvested. For example, in international investing, a company may be required to reinvest the cash flows in the country in which they are generated and not “repatriate” the money. Such funds are said to be “blocked” and reinvestment becomes relevant because the cash flows are not truly available.

14. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the IRR, then calculate the IRR of this future value and the initial investment, you will get the same IRR. However, as in the previous question, what is done with the cash flows once they are generated does not affect the IRR. Consider the following example:

	$C_0$	$C_1$	$C_2$	IRR
Project A	-\$100	\$10	\$110	10%

Suppose this \$100 is a deposit into a bank account. The IRR of the cash flows is 10 percent. Does the IRR change if the Year 1 cash flow is reinvested in the account, or if it is withdrawn and spent on pizza? No. Finally, consider the yield to maturity calculation on a bond. If you think about it, the YTM is the IRR on the bond, but no mention of a reinvestment assumption for the bond coupons is suggested. The reason is that reinvestment is irrelevant to the YTM calculation; in the same way, reinvestment is irrelevant in the IRR calculation. Our caveat about blocked funds applies here as well.

## Solutions to Questions and Problems

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

### Basic

1. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Project A:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$9,500 &&= \$9,500 \\ \text{Cumulative cash flows Year 2} &= \$9,500 + 6,000 &&= \$15,500 \end{aligned}$$

Companies can calculate a more precise value using fractional years. To calculate the fractional payback period, find the fraction of year 2's cash flows that is needed for the company to have cumulative undiscounted cash flows of \$15,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of year 1 by the undiscounted cash flow of year 2.

$$\text{Payback period} = 1 + (\$15,000 - 9,500) / \$6,000$$

$$\text{Payback period} = 1.917 \text{ years}$$

Project B:

$$\text{Cumulative cash flows Year 1} = \$10,500 = \$10,500$$

$$\text{Cumulative cash flows Year 2} = \$10,500 + 7,000 = \$17,500$$

$$\text{Cumulative cash flows Year 3} = \$10,500 + 7,000 + 6,000 = \$23,500$$

To calculate the fractional payback period, find the fraction of year 3's cash flows that is needed for the company to have cumulative undiscounted cash flows of \$18,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of year 2 by the undiscounted cash flow of year 3.

$$\text{Payback period} = 2 + (\$18,000 - 10,500 - 7,000) / \$6,000$$

$$\text{Payback period} = 2.083 \text{ years}$$

Since project A has a shorter payback period than project B has, the company should choose project A.

- b. Discount each project's cash flows at 15 percent. Choose the project with the highest NPV.

Project A:

$$\text{NPV} = -\$15,000 + \$9,500 / 1.15 + \$6,000 / 1.15^2 + \$2,400 / 1.15^3$$

$$\text{NPV} = -\$624.23$$

Project B:

$$\text{NPV} = -\$18,000 + \$10,500 / 1.15 + \$7,000 / 1.15^2 + \$6,000 / 1.15^3$$

$$\text{NPV} = \$368.54$$

The firm should choose Project B since it has a higher NPV than Project A has.

2. To calculate the payback period, we need to find the time that the project has taken to recover its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is \$3,400, the payback period is:

$$\text{Payback} = 3 + (\$680 / \$840) = 3.81 \text{ years}$$

There is a shortcut to calculate the payback period if the future cash flows are an annuity. Just divide the initial cost by the annual cash flow. For the \$3,400 cost, the payback period is:

$$\text{Payback} = \$3,400 / \$840 = 3.81 \text{ years}$$

For an initial cost of \$4,800, the payback period is:

$$\text{Payback} = \$4,800 / \$840 = 5.71 \text{ years}$$

The payback period for an initial cost of \$7,300 is a little trickier. Notice that the total cash inflows after eight years will be:

$$\text{Total cash inflows} = 8(\$840) = \$6,720$$

If the initial cost is \$7,300, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

$$\text{Payback} = \$7,300 / \$840 = 8.69 \text{ years}$$

This answer does not make sense since the cash flows stop after eight years, so there is no payback period.

3. When we use discounted payback, we need to find the value of all cash flows today. The value today of the project cash flows for the first four years is:

$$\text{Value today of Year 1 cash flow} = \$5,000/1.14 = \$4,385.96$$

$$\text{Value today of Year 2 cash flow} = \$5,500/1.14^2 = \$4,232.07$$

$$\text{Value today of Year 3 cash flow} = \$6,000/1.14^3 = \$4,049.83$$

$$\text{Value today of Year 4 cash flow} = \$7,000/1.14^4 = \$4,144.56$$

To find the discounted payback, we use these values to find the payback period. The discounted first year cash flow is \$4,385.96, so the discounted payback for an initial cost of \$8,000 is:

$$\text{Discounted payback} = 1 + (\$8,000 - 4,385.96)/\$4,232.07 = 1.85 \text{ years}$$

For an initial cost of \$12,000, the discounted payback is:

$$\text{Discounted payback} = 2 + (\$12,000 - 4,385.96 - 4,232.07)/\$4,049.83 = 2.84 \text{ years}$$

Notice the calculation of discounted payback. We know the payback period is between two and three years, so we subtract the discounted values of the Year 1 and Year 2 cash flows from the initial cost. This is the numerator, which is the discounted amount we still need to make to recover our initial investment. We divide this amount by the discounted amount we will earn in Year 3 to get the fractional portion of the discounted payback.

If the initial cost is \$16,000, the discounted payback is:

$$\text{Discounted payback} = 3 + (\$16,000 - 4,385.96 - 4,232.07 - 4,049.83) / \$4,144.56 = 3.80 \text{ years}$$

4. To calculate the discounted payback, discount all future cash flows back to the present, and use these discounted cash flows to calculate the payback period. To find the fractional year, we divide the amount we need to make in the last year to payback the project by the amount we will make. Doing so, we find:

$$R = 0\%: 3 + (\$3,600 / \$3,800) = 3.95 \text{ years}$$

$$\text{Discounted payback} = \text{Regular payback} = 3.95 \text{ years}$$

$$R = 10\%: \$3,800/1.10 + \$3,800/1.10^2 + \$3,800/1.10^3 + \$3,800/1.10^4 + \$3,800/1.10^5 = \$14,404.99$$

$$\$3,800/1.10^6 = \$2,145.00$$

$$\text{Discounted payback} = 5 + (\$15,000 - 14,404.99) / \$2,145.00 = 5.28 \text{ years}$$

$$R = 15\%: \$3,800/1.15 + \$3,800/1.15^2 + \$3,800/1.15^3 + \$3,800/1.15^4 + \$3,800/1.15^5 + \$3,800/1.15^6$$

$$= \$14,381.03; \text{ The project never pays back.}$$

5. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$20,000 + \$8,500/(1 + \text{IRR}) + \$10,200/(1 + \text{IRR})^2 + \$6,200/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 12.41\%$$

Since the IRR is greater than the required return we would accept the project.

6. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this Project A is:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$5,300 + \$2,000/(1 + \text{IRR}) + \$2,800/(1 + \text{IRR})^2 + \$1,600/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 10.38\%$$

And the IRR for Project B is:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$2,900 + \$1,100/(1 + \text{IRR}) + \$1,800/(1 + \text{IRR})^2 + \$1,200/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 19.16\%$$

7. The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The cash flows from this project are an annuity, so the equation for the profitability index is:

$$\text{PI} = C(\text{PVIFA}_{R,t}) / C_0$$

$$\text{PI} = \$84,000(\text{PVIFA}_{13\%,7}) / \$385,000$$

$$\text{PI} = 0.965$$

8. a. The profitability index is the present value of the future cash flows divided by the initial cost. So, for Project Alpha, the profitability index is:

$$PI_{\text{Alpha}} = [\$1,200 / 1.10 + \$1,100 / 1.10^2 + \$900 / 1.10^3] / \$2,300 = 1.164$$

And for Project Beta the profitability index is:

$$PI_{\text{Beta}} = [\$800 / 1.10 + \$2,300 / 1.10^2 + \$2,900 / 1.10^3] / \$3,900 = 1.233$$

- b. According to the profitability index, you would accept Project Beta. However, remember the profitability index rule can lead to an incorrect decision when ranking mutually exclusive projects.

### Intermediate

9. a. To have a payback equal to the project's life, given  $C$  is a constant cash flow for  $N$  years:

$$C = I/N$$

- b. To have a positive NPV,  $I < C (PVIFA_{R\%, N})$ . Thus,  $C > I / (PVIFA_{R\%, N})$ .

- c. Benefit =  $C (PVIFA_{R\%, N}) = 2 \times \text{costs} = 2I$   
 $C = 2I / (PVIFA_{R\%, N})$

10. a. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$$\begin{aligned} 0 &= C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3 + C_4 / (1 + \text{IRR})^4 \\ 0 &= \$7,000 - \$3,700 / (1 + \text{IRR}) - \$2,400 / (1 + \text{IRR})^2 - \$1,500 / (1 + \text{IRR})^3 \\ &\quad - \$1,200 / (1 + \text{IRR})^4 \end{aligned}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 12.40\%$$

- b. This problem differs from previous ones because the initial cash flow is positive and all future cash flows are negative. In other words, this is a financing-type project, while previous projects were investing-type projects. For financing situations, accept the project when the IRR is less than the discount rate. Reject the project when the IRR is greater than the discount rate.

$$\text{IRR} = 12.40\%$$

$$\text{Discount Rate} = 10\%$$

$$\text{IRR} > \text{Discount Rate}$$

Reject the offer when the discount rate is less than the IRR.



- c. Using the same reason as part b., we would accept the project if the discount rate is 20 percent.

$$\text{IRR} = 12.40\%$$

$$\text{Discount Rate} = 20\%$$

$$\text{IRR} < \text{Discount Rate}$$

Accept the offer when the discount rate is greater than the IRR.

- d. The NPV is the sum of the present value of all cash flows, so the NPV of the project if the discount rate is 10 percent will be:

$$\text{NPV} = \$7,000 - \$3,700 / 1.1 - \$2,400 / 1.1^2 - \$1,500 / 1.1^3 - \$1,200 / 1.1^4$$

$$\text{NPV} = -\$293.70$$

When the discount rate is 10 percent, the NPV of the offer is  $-\$293.70$ . Reject the offer.

And the NPV of the project if the discount rate is 20 percent will be:

$$\text{NPV} = \$7,000 - \$3,700 / 1.2 - \$2,400 / 1.2^2 - \$1,500 / 1.2^3 - \$1,200 / 1.2^4$$

$$\text{NPV} = \$803.24$$

When the discount rate is 20 percent, the NPV of the offer is  $\$803.24$ . Accept the offer.

- e. Yes, the decisions under the NPV rule are consistent with the choices made under the IRR rule since the signs of the cash flows change only once.

11. a. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR for each project is:

Deepwater Fishing IRR:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$950,000 + \$370,000 / (1 + \text{IRR}) + \$510,000 / (1 + \text{IRR})^2 + \$420,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 17.07\%$$

Submarine Ride IRR:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$1,850,000 + \$900,000 / (1 + \text{IRR}) + \$800,000 / (1 + \text{IRR})^2 + \$750,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 16.03\%$$

Based on the IRR rule, the deepwater fishing project should be chosen because it has the higher IRR.

- b. To calculate the incremental IRR, we subtract the smaller project's cash flows from the larger project's cash flows. In this case, we subtract the deepwater fishing cash flows from the submarine ride cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the submarine ride are:

	Year 0	Year 1	Year 2	Year 3
Submarine Ride	-\$1,850,000	\$900,000	\$800,000	\$750,000
Deepwater Fishing	-950,000	370,000	510,000	420,000
Submarine – Fishing	-\$900,000	\$530,000	\$290,000	\$330,000

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$900,000 + \$530,000 / (1 + \text{IRR}) + \$290,000 / (1 + \text{IRR})^2 + \$330,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{Incremental IRR} = 14.79\%$$

For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 14.79%, is greater than the required rate of return of 14 percent, choose the submarine ride project. Note that this is not the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the submarine ride has a greater initial investment than does the deepwater fishing project. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

- c. The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

Deepwater fishing:

$$\text{NPV} = -\$950,000 + \$370,000 / 1.14 + \$510,000 / 1.14^2 + \$420,000 / 1.14^3$$

$$\text{NPV} = \$50,477.88$$

Submarine ride:

$$\text{NPV} = -\$1,850,000 + \$900,000 / 1.14 + \$800,000 / 1.14^2 + \$750,000 / 1.14^3$$

$$\text{NPV} = \$61,276.34$$

Since the NPV of the submarine ride project is greater than the NPV of the deepwater fishing project, choose the submarine ride project. The incremental IRR rule is always consistent with the NPV rule.

12. a. The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

$$PI_I = \$18,000(PVIFA_{10\%,3}) / \$30,000 = 1.492$$

$$PI_{II} = \$7,500(PVIFA_{10\%,3}) / \$12,000 = 1.554$$

The profitability index decision rule implies that we accept project II, since  $PI_{II}$  is greater than the  $PI_I$ .

- b. The NPV of each project is:

$$NPV_I = -\$30,000 + \$18,000(PVIFA_{10\%,3}) = \$14,763.34$$

$$NPV_{II} = -\$12,000 + \$7,500(PVIFA_{10\%,3}) = \$6,651.39$$

The NPV decision rule implies accepting Project I, since the  $NPV_I$  is greater than the  $NPV_{II}$ .

- c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitudes of the cash flows for the two projects are of different scales. In this problem, project I is 2.5 times as large as project II and produces a larger NPV, yet the profitability index criterion implies that project II is more acceptable.

13. a. The equation for the NPV of the project is:

$$NPV = -\$85,000,000 + \$125,000,000/1.1 - \$15,000,000/1.1^2 = \$16,239,669.42$$

The NPV is greater than 0, so we would accept the project.

- b. The equation for the IRR of the project is:

$$0 = -\$85,000,000 + \$125,000,000/(1+IRR) - \$15,000,000/(1+IRR)^2$$

From Descartes' rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

$$IRR = 33.88\%, -86.82\%$$

When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct; that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

14. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Board game:

$$\text{Cumulative cash flows Year 1} = \$600 \quad = \$600$$

$$\text{Cumulative cash flows Year 2} = \$600 + 450 \quad = \$1,050$$

$$\text{Payback period} = 1 + \$150 / \$450 \quad = 1.33 \text{ years}$$

DVD:

$$\text{Cumulative cash flows Year 1} = \$1,300 = \$1,300$$

$$\text{Cumulative cash flows Year 2} = \$1,300 + 850 = \$2,150$$

$$\text{Payback period} = 1 + (\$1,800 - 1,300) / \$850$$

$$\text{Payback period} = 1.59 \text{ years}$$

Since the board game has a shorter payback period than the DVD project, the company should choose the board game.

- b. The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

Board game:

$$\text{NPV} = -\$750 + \$600 / 1.10 + \$450 / 1.10^2 + \$120 / 1.10^3$$

$$\text{NPV} = \$257.51$$

DVD:

$$\text{NPV} = -\$1,850 + \$1,300 / 1.10 + \$850 / 1.10^2 + \$350 / 1.10^3$$

$$\text{NPV} = \$347.26$$

Since the NPV of the DVD is greater than the NPV of the board game, choose the DVD.

- c. The IRR is the interest rate that makes the NPV of a project equal to zero. So, the IRR of each project is:

Board game:

$$0 = -\$750 + \$600 / (1 + \text{IRR}) + \$450 / (1 + \text{IRR})^2 + \$120 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 33.79\%$$

DVD:

$$0 = -\$1,850 + \$1,300 / (1 + \text{IRR}) + \$850 / (1 + \text{IRR})^2 + \$350 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 23.31\%$$

Since the IRR of the board game is greater than the IRR of the DVD, IRR implies we choose the board game. Note that this is the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the DVD has a greater initial investment than does the board game. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

- d. To calculate the incremental IRR, we subtract the smaller project's cash flows from the larger project's cash flows. In this case, we subtract the board game cash flows from the DVD cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the DVD are:

	Year 0	Year 1	Year 2	Year 3
DVD	-\$1,800	\$1,300	\$850	\$350
Board game	-750	600	450	120
DVD – Board game	-\$1,050	\$700	\$400	\$230

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

$$0 = C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3$$

$$0 = -\$1,050 + \$700 / (1 + \text{IRR}) + \$400 / (1 + \text{IRR})^2 + \$230 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{Incremental IRR} = 15.86\%$$

For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 15.86%, is greater than the required rate of return of 10 percent, choose the DVD project.

15. a. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

$$\text{PI}_{\text{CDMA}} = [\$11,000,000 / 1.10 + \$7,500,000 / 1.10^2 + \$2,500,000 / 1.10^3] / \$8,000,000 = 2.26$$

$$\text{PI}_{\text{G4}} = [\$10,000,000 / 1.10 + \$25,000,000 / 1.10^2 + \$20,000,000 / 1.10^3] / \$12,000,000 = 3.73$$

$$\text{PI}_{\text{Wi-Fi}} = [\$18,000,000 / 1.10 + \$32,000,000 / 1.10^2 + \$20,000,000 / 1.10^3] / \$20,000,000 = 2.89$$

The profitability index implies we accept the G4 project. Remember this is not necessarily correct because the profitability index does not necessarily rank projects with different initial investments correctly.

- b. The NPV of each project is:

$$\text{NPV}_{\text{CDMA}} = -\$8,000,000 + \$11,000,000 / 1.10 + \$7,500,000 / 1.10^2 + \$2,500,000 / 1.10^3$$

$$\text{NPV}_{\text{CDMA}} = \$10,076,634.11$$

$$\text{NPV}_{\text{G4}} = -\$12,000,000 + \$10,000,000 / 1.10 + \$25,000,000 / 1.10^2 + \$20,000,000 / 1.10^3$$

$$\text{NPV}_{\text{G4}} = \$32,778,362.13$$

$$\begin{aligned} \text{NPV}_{\text{Wi-Fi}} &= -\$20,000,000 + \$18,000,000 / 1.10 + \$32,000,000 / 1.10^2 + \$20,000,000 / 1.10^3 \\ \text{NPV}_{\text{Wi-Fi}} &= \$37,836,213.37 \end{aligned}$$

NPV implies we accept the Wi-Fi project since it has the highest NPV. This is the correct decision if the projects are mutually exclusive.

- c. We would like to invest in all three projects since each has a positive NPV. If the budget is limited to \$20 million, we can only accept the CDMA project and the G4 project, or the Wi-Fi project. NPV is additive across projects and the company. The total NPV of the CDMA project and the G4 project is:

$$\begin{aligned} \text{NPV}_{\text{CDMA and G4}} &= \$10,076,634.11 + 32,778,362.13 \\ \text{NPV}_{\text{CDMA and G4}} &= \$42,854,996.24 \end{aligned}$$

This is greater than the Wi-Fi project, so we should accept the CDMA project and the G4 project.

16. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

AZM Mini-SUV:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$320,000 &&= \$320,000 \\ \text{Cumulative cash flows Year 2} &= \$320,000 + 180,000 &&= \$500,000 \end{aligned}$$

$$\text{Payback period} = 1 + \$130,000 / \$180,000 = 1.72 \text{ years}$$

AZF Full-SUV:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$350,000 &&= \$350,000 \\ \text{Cumulative cash flows Year 2} &= \$350,000 + 420,000 &&= \$770,000 \\ \text{Cumulative cash flows Year 2} &= \$350,000 + 420,000 + 290,000 &&= \$1,060,000 \end{aligned}$$

$$\text{Payback period} = 2 + \$30,000 / \$290,000 = 2.10 \text{ years}$$

Since the AZM has a shorter payback period than the AZF, the company should choose the AZM. Remember the payback period does not necessarily rank projects correctly.

- b. The NPV of each project is:

$$\begin{aligned} \text{NPV}_{\text{AZM}} &= -\$450,000 + \$320,000 / 1.10 + \$180,000 / 1.10^2 + \$150,000 / 1.10^3 \\ \text{NPV}_{\text{AZM}} &= \$102,366.64 \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{AZF}} &= -\$800,000 + \$350,000 / 1.10 + \$420,000 / 1.10^2 + \$290,000 / 1.10^3 \\ \text{NPV}_{\text{AZF}} &= \$83,170.55 \end{aligned}$$

The NPV criteria implies we accept the AZM because it has the highest NPV.

- c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the AZM is:

$$0 = -\$450,000 + \$320,000 / (1 + \text{IRR}) + \$180,000 / (1 + \text{IRR})^2 + \$150,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{AZM}} = 24.65\%$$

And the IRR of the AZF is:

$$0 = -\$800,000 + \$350,000 / (1 + \text{IRR}) + \$420,000 / (1 + \text{IRR})^2 + \$290,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{AZF}} = 15.97\%$$

The IRR criteria implies we accept the AZM because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

- d. Incremental IRR analysis is not necessary. The AZM has the smallest initial investment, and the largest NPV, so it should be accepted.

17. a. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

$$\text{PI}_A = [\$110,000 / 1.12 + \$110,000 / 1.12^2] / \$150,000 = 1.24$$

$$\text{PI}_B = [\$200,000 / 1.12 + \$200,000 / 1.12^2] / \$300,000 = 1.13$$

$$\text{PI}_C = [\$120,000 / 1.12 + \$90,000 / 1.12^2] / \$150,000 = 1.19$$

- b. The NPV of each project is:

$$\begin{aligned} \text{NPV}_A &= -\$150,000 + \$110,000 / 1.12 + \$110,000 / 1.12^2 \\ \text{NPV}_A &= \$35,905.61 \end{aligned}$$

$$\begin{aligned} \text{NPV}_B &= -\$300,000 + \$200,000 / 1.12 + \$200,000 / 1.12^2 \\ \text{NPV}_B &= \$38,010.20 \end{aligned}$$

$$\begin{aligned} \text{NPV}_C &= -\$150,000 + \$120,000 / 1.12 + \$90,000 / 1.12^2 \\ \text{NPV}_C &= \$28,890.31 \end{aligned}$$

- c. Accept projects A, B, and C. Since the projects are independent, accept all three projects because the respective profitability index of each is greater than one.

- d. Accept Project B. Since the Projects are mutually exclusive, choose the Project with the highest PI, while taking into account the scale of the Project. Because Projects A and C have the same initial investment, the problem of scale does not arise when comparing the profitability indices. Based on the profitability index rule, Project C can be eliminated because its PI is less than the PI of Project A. Because of the problem of scale, we cannot compare the PIs of Projects A and B. However, we can calculate the PI of the incremental cash flows of the two projects, which are:

Project	$C_0$	$C_1$	$C_2$
B – A	–\$150,000	\$90,000	\$90,000

When calculating incremental cash flows, remember to subtract the cash flows of the project with the smaller initial cash outflow from those of the project with the larger initial cash outflow. This procedure insures that the incremental initial cash outflow will be negative. The incremental PI calculation is:

$$\text{PI}(B - A) = [\$90,000 / 1.12 + \$90,000 / 1.12^2] / \$150,000$$

$$\text{PI}(B - A) = 1.014$$

The company should accept Project B since the PI of the incremental cash flows is greater than one.

- e. Remember that the NPV is additive across projects. Since we can spend \$450,000, we could take two of the projects. In this case, we should take the two projects with the highest NPVs, which are Project B and Project A.
18. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Dry Prepeg:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$1,100,000 &&= \$1,100,000 \\ \text{Cumulative cash flows Year 2} &= \$1,100,000 + 900,000 &&= \$2,000,000 \end{aligned}$$

$$\text{Payback period} = 1 + (\$600,000 / \$900,000) = 1.67 \text{ years}$$

Solvent Prepeg:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$375,000 &&= \$375,000 \\ \text{Cumulative cash flows Year 2} &= \$375,000 + 600,000 &&= \$975,000 \end{aligned}$$

$$\text{Payback period} = 1 + (\$375,000 / \$600,000) = 1.63 \text{ years}$$

Since the solvent prepeg has a shorter payback period than the dry prepeg, the company should choose the solvent prepeg. Remember the payback period does not necessarily rank projects correctly.



b. The NPV of each project is:

$$\begin{aligned} \text{NPV}_{\text{Dry prepeg}} &= -\$1,700,000 + \$1,100,000 / 1.10 + \$900,000 / 1.10^2 + \$750,000 / 1.10^3 \\ \text{NPV}_{\text{Dry prepeg}} &= \$607,287.75 \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{Solvent prepeg}} &= -\$750,000 + \$375,000 / 1.10 + \$600,000 / 1.10^2 + \$390,000 / 1.10^3 \\ \text{NPV}_{\text{Solvent prepeg}} &= \$379,789.63 \end{aligned}$$

The NPV criteria implies accepting the dry prepeg because it has the highest NPV.

c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the dry prepeg is:

$$0 = -\$1,700,000 + \$1,100,000 / (1 + \text{IRR}) + \$900,000 / (1 + \text{IRR})^2 + \$750,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{Dry prepeg}} = 30.90\%$$

And the IRR of the solvent prepeg is:

$$0 = -\$750,000 + \$375,000 / (1 + \text{IRR}) + \$600,000 / (1 + \text{IRR})^2 + \$390,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{Solvent prepeg}} = 36.51\%$$

The IRR criteria implies accepting the solvent prepeg because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

d. Incremental IRR analysis is necessary. The solvent prepeg has a higher IRR, but is relatively smaller in terms of investment and NPV. In calculating the incremental cash flows, we subtract the cash flows from the project with the smaller initial investment from the cash flows of the project with the large initial investment, so the incremental cash flows are:

	Year 0	Year 1	Year 2	Year 3
Dry prepeg	-\$1,700,000	\$1,100,000	\$900,000	\$750,000
Solvent prepeg	-750,000	375,000	600,000	390,000
Dry prepeg – Solvent prepeg	-\$950,000	\$725,000	\$300,000	\$360,000

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

$$0 = -\$950,000 + \$725,000 / (1 + \text{IRR}) + \$300,000 / (1 + \text{IRR})^2 + \$360,000 / (1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{Incremental IRR} = 25.52\%$$

For investing-type projects, we accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 25.52%, is greater than the required rate of return of 10 percent, we choose the dry prepeg.

19. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

NP-30:

Cumulative cash flows Year 1 = \$185,000	= \$185,000
Cumulative cash flows Year 2 = \$185,000 + 185,000	= \$370,000
Cumulative cash flows Year 3 = \$185,000 + 185,000 + 185,000	= \$555,000

Payback period = 2 + (\$180,000/\$185,000) = 2.97 years

NX-20:

Cumulative cash flows Year 1 = \$100,000	= \$100,000
Cumulative cash flows Year 2 = \$100,000 + 110,000	= \$210,000
Cumulative cash flows Year 3 = \$100,000 + 110,000 + 121,000	= \$331,000
Cumulative cash flows Year 4 = \$100,000 + 110,000 + 121,000 + 133,100	= \$464,100

Payback period = 3 + (\$19,000/\$133,100) = 3.14 years

Since the NP-30 has a shorter payback period than the NX-20, the company should choose the NP-30. Remember the payback period does not necessarily rank projects correctly.

- b. The IRR is the interest rate that makes the NPV of the project equal to zero, so the IRR of each project is:

NP-30:

$$0 = -\$550,000 + \$185,000 \left( \frac{1 - [1/(1 + \text{IRR})]^5}{\text{IRR}} \right)$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{NP-30}} = 20.27\%$$

And the IRR of the NX-20 is:

$$0 = -\$350,000 + \$100,000 / (1 + \text{IRR}) + \$110,000 / (1 + \text{IRR})^2 + \$121,000 / (1 + \text{IRR})^3 + \$133,100 / (1 + \text{IRR})^4 + \$146,410 / (1 + \text{IRR})^5$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$IRR_{NX-20} = 20.34\%$$

The IRR criteria implies accepting the NX-20.

- c. The profitability index is the present value of all subsequent cash flows, divided by the initial investment, so the profitability index of each project is:

$$PI_{NP-30} = (\$185,000 \{ [1 - (1/1.15)^5] / .15 \}) / \$550,000$$

$$PI_{NP-30} = 1.128$$

$$PI_{NX-20} = [\$100,000 / 1.15 + \$110,000 / 1.15^2 + \$121,000 / 1.15^3 + \$133,100 / 1.15^4 + \$146,410 / 1.15^5] / \$350,000$$

$$PI_{NX-20} = 1.139$$

The PI criteria implies accepting the NX-20.

- d. The NPV of each project is:

$$NPV_{NP-30} = -\$550,000 + \$185,000 \{ [1 - (1/1.15)^5] / .15 \}$$

$$NPV_{NP-30} = \$70,148.69$$

$$NPV_{NX-20} = -\$350,000 + \$100,000 / 1.15 + \$110,000 / 1.15^2 + \$121,000 / 1.15^3 + \$133,100 / 1.15^4 + \$146,410 / 1.15^5$$

$$NPV_{NX-20} = \$48,583.79$$

The NPV criteria implies accepting the NP-30.

### Challenge

20. The equation for the IRR of the project is:

$$0 = -\$75,000 + \$155,000/(1+IRR) - \$65,000/(1+IRR)^2$$

From Descartes' Rule of Signs, we know there are either zero IRRs or two IRRs since the cash flows change signs twice. We can rewrite this equation as:

$$0 = -\$75,000 + \$155,000X - \$65,000X^2$$

where  $X = 1 / (1 + IRR)$

This is a quadratic equation. We can solve for the roots of this equation with the quadratic formula:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remember that the quadratic formula is written as:

$$0 = aX^2 + bX + c$$

In this case, the equation is:

$$0 = -\$65,000X^2 + \$155,000X - \$75,000$$

$$X = \frac{-155,000 \pm \sqrt{(155,000)^2 - 4(-75,000)(-65,000)}}{2(-65,000)}$$

$$X = \frac{-155,000 \pm \sqrt{4,525,000,000}}{2(-65,000)}$$

$$X = \frac{-155,000 \pm 67,268.12}{-130,000}$$

Solving the quadratic equation, we find two Xs:

$$X = 0.6749, 1.7098$$

Since:

$$X = 1 / (1 + \text{IRR})$$

$$1.7098 = 1 / (1 + \text{IRR})$$

$$\text{IRR} = -.4151 \text{ or } -41.51\%$$

And:

$$X = 1 / (1 + \text{IRR})$$

$$0.6749 = 1 / (1 + \text{IRR})$$

$$\text{IRR} = 0.4818 \text{ or } 48.18\%$$

To find the maximum (or minimum) of a function, we find the derivative and set it equal to zero.  
The derivative of this IRR function is:

$$0 = -\$155,000(1 + \text{IRR})^{-2} + \$130,000(1 + \text{IRR})^{-3}$$

$$-\$155,000(1 + \text{IRR})^{-2} = \$130,000(1 + \text{IRR})^{-3}$$

$$-\$155,000(1 + \text{IRR})^3 = \$130,000(1 + \text{IRR})^2$$

$$-\$155,000(1 + \text{IRR}) = \$130,000$$

$$\text{IRR} = \$130,000 / \$155,000 - 1$$

$$\text{IRR} = -.1613 \text{ or } -16.13\%$$

To determine if this is a maximum or minimum, we can find the second derivative of the IRR function. If the second derivative is positive, we have found a minimum and if the second derivative is negative we have found a maximum. Using the reduced equation above, that is:

$$-\$155,000(1 + \text{IRR}) = \$130,000$$

The second derivative is  $-\$262,722.18$ , therefore we have a maximum.

- 21.** Given the six-year payback, the worst case is that the payback occurs at the end of the sixth year. Thus, the worst case:

$$\text{NPV} = -\$434,000 + \$434,000/1.12^6 = -\$214,122.09$$

The best case has infinite cash flows beyond the payback point. Thus, the best-case NPV is infinite.

- 22.** The equation for the IRR of the project is:

$$0 = -\$1,008 + \$5,724/(1 + \text{IRR}) - \$12,140/(1 + \text{IRR})^2 + \$11,400/(1 + \text{IRR})^3 - \$4,000/(1 + \text{IRR})^4$$

Using Descartes' rule of signs, from looking at the cash flows we know there are four IRRs for this project. Even with most computer spreadsheets, we have to do some trial and error. From trial and error, IRRs of 25%, 33.33%, 42.86%, and 66.67% are found.

We would accept the project when the NPV is greater than zero. See for yourself that the NPV is greater than zero for required returns between 25% and 33.33% or between 42.86% and 66.67%.

- 23. a.** Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. The PV of the future cash flows from the project is:

$$\begin{aligned} \text{PV of cash inflows} &= C_1/(R - g) \\ \text{PV of cash inflows} &= \$290,000/ (.11 - .05) = \$4,833,333.33 \end{aligned}$$

NPV is the PV of the outflows minus by the PV of the inflows, so the NPV is:

$$\text{NPV of the project} = -\$3,900,000 + 4,833,333.33 = \$933,333.33$$

The NPV is positive, so we would accept the project.

- b.** Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:

$$0 = -\$3,900,000 + \$290,000/ (.11 - g)$$

Solving for  $g$ , we get:

$$g = 3.56\%$$

24. a. The project involves three cash flows: the initial investment, the annual cash inflows, and the abandonment costs. The mine will generate cash inflows over its 11-year economic life. To express the PV of the annual cash inflows, apply the growing annuity formula, discounted at the IRR and growing at eight percent.

$$\begin{aligned} \text{PV}(\text{Cash Inflows}) &= C \{ [1/(r-g)] - [1/(r-g)] \times [(1+g)/(1+r)]^t \} \\ \text{PV}(\text{Cash Inflows}) &= \$345,000 \{ [1/(\text{IRR} - .08)] - [1/(\text{IRR} - .08)] \times [(1 + .08)/(1 + \text{IRR})]^{11} \} \end{aligned}$$

At the end of 11 years, the company will abandon the mine, incurring a \$400,000 charge. Discounting the abandonment costs back 11 years at the IRR to express its present value, we get:

$$\begin{aligned} \text{PV}(\text{Abandonment}) &= C_{11} / (1 + \text{IRR})^{11} \\ \text{PV}(\text{Abandonment}) &= -\$400,000 / (1 + \text{IRR})^{11} \end{aligned}$$

So, the IRR equation for this project is:

$$0 = -\$2,400,000 + \$345,000 \{ [1/(\text{IRR} - .08)] - [1/(\text{IRR} - .08)] \times [(1 + .08)/(1 + \text{IRR})]^{11} \} - \$400,000 / (1 + \text{IRR})^{11}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 14.74\%$$

- b. Yes. Since the mine's IRR exceeds the required return of 10 percent, the mine should be opened. The correct decision rule for an investment-type project is to accept the project if the IRR is greater than the discount rate. Although it appears there is a sign change at the end of the project because of the abandonment costs, the last cash flow is actually positive because of the operating cash flow in the last year.
25. First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

$$\begin{aligned} \text{Year 2 cash flow} &= \$285,000(1.04) = \$296,400 \\ \text{Year 3 cash flow} &= \$345,000(1.04) = \$358,800 \\ \text{Year 4 cash flow} &= \$415,000(1.04) = \$431,600 \\ \text{Year 5 cash flow} &= \$255,000(1.04) = \$265,200 \end{aligned}$$

So, the NPV of the project is:

$$\begin{aligned} \text{NPV} &= -\$950,000 + \$296,400/1.11^2 + \$358,800/1.11^3 + \$431,600/1.11^4 + \$265,200/1.11^5 \\ \text{NPV} &= -\$5,392.06 \end{aligned}$$

And the IRR of the project is:

$$0 = -\$950,000 + \$296,400/(1 + \text{IRR})^2 + \$358,800/(1 + \text{IRR})^3 + \$431,600/(1 + \text{IRR})^4 + \$265,200/(1 + \text{IRR})^5$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 10.81\%$$

While this may look like a MIRR calculation, it is not a MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

26. a. We can apply the growing perpetuity formula to find the PV of stream A. The perpetuity formula values the stream as of one year before the first payment. Therefore, the growing perpetuity formula values the stream of cash flows as of year 2. Next, discount the PV as of the end of year 2 back two years to find the PV as of today, year 0. Doing so, we find:

$$\begin{aligned} \text{PV}(A) &= [C_3 / (R - g)] / (1 + R)^2 \\ \text{PV}(A) &= [\$8,900 / (0.12 - 0.04)] / (1.12)^2 \\ \text{PV}(A) &= \$88,687.82 \end{aligned}$$

We can apply the perpetuity formula to find the PV of stream B. The perpetuity formula discounts the stream back to year 1, one period prior to the first cash flow. Discount the PV as of the end of year 1 back one year to find the PV as of today, year 0. Doing so, we find:

$$\begin{aligned} \text{PV}(B) &= [C_2 / R] / (1 + R) \\ \text{PV}(B) &= [-\$10,000 / 0.12] / (1.12) \\ \text{PV}(B) &= -\$74,404.76 \end{aligned}$$

b. If we combine the cash flow streams to form Project C, we get:

$$\text{Project A} = [C_3 / (R - G)] / (1 + R)^2$$

$$\text{Project B} = [C_2 / R] / (1 + R)$$

$$\text{Project C} = \text{Project A} + \text{Project B}$$

$$\text{Project C} = [C_3 / (R - g)] / (1 + R)^2 + [C_2 / R] / (1 + R)$$

$$0 = [\$8,900 / (IRR - .04)] / (1 + IRR)^2 + [-\$10,000 / IRR] / (1 + IRR)$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 16.80\%$$

c. The correct decision rule for an investing-type project is to accept the project if the discount rate is below the IRR. Since there is one IRR, a decision can be made. At a point in the future, the cash flows from stream A will be greater than those from stream B. Therefore, although there are many cash flows, there will be only one change in sign. When the sign of the cash flows change more than once over the life of the project, there may be multiple internal rates of return. In such cases, there is no correct decision rule for accepting and rejecting projects using the internal rate of return.

27. To answer this question, we need to examine the incremental cash flows. To make the projects equally attractive, Project Billion must have a larger initial investment. We know this because the subsequent cash flows from Project Billion are larger than the subsequent cash flows from Project Million. So, subtracting the Project Million cash flows from the Project Billion cash flows, we find the incremental cash flows are:

<u>Year</u>	<u>Incremental cash flows</u>
0	$-I_0 + \$1,200$
1	240
2	240
3	400

Now we can find the present value of the subsequent incremental cash flows at the discount rate, 12 percent. The present value of the incremental cash flows is:

$$\text{PV} = \$1,200 + \$240 / 1.12 + \$240 / 1.12^2 + \$400 / 1.12^3$$

$$\text{PV} = \$1,890.32$$

So, if  $I_0$  is greater than \$1,890.32, the incremental cash flows will be negative. Since we are subtracting Project Million from Project Billion, this implies that for any value over \$1,890.32 the NPV of Project Billion will be less than that of Project Million, so  $I_0$  must be less than \$1,890.32.



28. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:

$$0 = \$20,000 - \$26,000 / (1 + \text{IRR}) + \$13,000 / (1 + \text{IRR})^2$$

Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don't remember, the quadratic equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, the equation is:

$$x = \frac{-(-26,000) \pm \sqrt{(-26,000)^2 - 4(20,000)(13,000)}}{2(20,000)}$$

The square root term works out to be:

$$676,000,000 - 1,040,000,000 = -364,000,000$$

The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.

## Calculator Solutions

1. b. *Project A*

<b>CF<sub>0</sub></b>	-\$15,000
<b>C01</b>	\$9,500
<b>F01</b>	1
<b>C02</b>	\$6,000
<b>F02</b>	1
<b>C03</b>	\$2,400
<b>F03</b>	1

I = 15%  
NPV CPT  
-\$624.23

<b>CF<sub>0</sub></b>	-\$18,000
<b>C01</b>	\$10,500
<b>F01</b>	1
<b>C02</b>	\$7,000
<b>F02</b>	1
<b>C03</b>	\$6,000
<b>F03</b>	1

I = 15%  
NPV CPT  
\$368.54

5.

<b>CF<sub>0</sub></b>	-\$20,000
<b>C01</b>	\$8,500
<b>F01</b>	1
<b>C02</b>	\$10,200
<b>F02</b>	1
<b>C03</b>	\$6,200
<b>F03</b>	1

IRR CPT  
12.41%

6. *Project A*

<b>CF<sub>0</sub></b>	-\$5,300
<b>C01</b>	\$2,000
<b>F01</b>	1
<b>C02</b>	\$2,800
<b>F02</b>	1
<b>C03</b>	\$1,600
<b>F03</b>	1

IRR CPT  
10.38%

*Project B*

<b>CF<sub>0</sub></b>	-\$2,900
<b>C01</b>	\$1,100
<b>F01</b>	1
<b>C02</b>	\$1,800
<b>F02</b>	1
<b>C03</b>	\$1,200
<b>F03</b>	1

IRR CPT  
19.16%

7.

<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$84,000
<b>F01</b>	7

I = 13%  
NPV CPT  
\$371,499.28

$$PI = \$371,499.28 / \$385,000 = 0.965$$

10.

<b>CF<sub>0</sub></b>	\$7,000
<b>C01</b>	-\$3,700
<b>F01</b>	1
<b>C02</b>	-\$2,400
<b>F02</b>	1
<b>C03</b>	-\$1,500
<b>F03</b>	1
<b>C04</b>	-\$1,200
<b>F04</b>	1

IRR CPT  
12.40%

<b>CF<sub>0</sub></b>	\$7,000
<b>C01</b>	-\$3,700
<b>F01</b>	1
<b>C02</b>	-\$2,400
<b>F02</b>	1
<b>C03</b>	-\$1,500
<b>F03</b>	1
<b>C04</b>	-\$1,200
<b>F04</b>	1

I = 10%  
NPV CPT  
-\$293.70

<b>CF<sub>0</sub></b>	\$7,000
<b>C01</b>	-\$3,700
<b>F01</b>	1
<b>C02</b>	-\$2,400
<b>F02</b>	1
<b>C03</b>	-\$1,500
<b>F03</b>	1
<b>C04</b>	-\$1,200
<b>F04</b>	1

I = 20%  
NPV CPT  
\$803.24

11. a.

*Deepwater fishing*

<b>CF<sub>0</sub></b>	-\$950,000
<b>C01</b>	\$370,000
<b>F01</b>	1
<b>C02</b>	\$510,000
<b>F02</b>	1
<b>C03</b>	\$420,000
<b>F03</b>	1

IRR CPT  
17.07%

*Submarine ride*

<b>CF<sub>0</sub></b>	-\$1,850,000
<b>C01</b>	\$900,000
<b>F01</b>	1
<b>C02</b>	\$800,000
<b>F02</b>	1
<b>C03</b>	\$750,000
<b>F03</b>	1

IRR CPT  
16.03%

b.

<b>CF<sub>0</sub></b>	-\$900,000
<b>C01</b>	\$530,000
<b>F01</b>	1
<b>C02</b>	\$290,000
<b>F02</b>	1
<b>C03</b>	\$330,000
<b>F03</b>	1
IRR CPT	
14.79%	

c.

*Deepwater fishing*

<b>CF<sub>0</sub></b>	-\$950,000
<b>C01</b>	\$370,000
<b>F01</b>	1
<b>C02</b>	\$510,000
<b>F02</b>	1
<b>C03</b>	\$420,000
<b>F03</b>	1
I = 14%	
NPV CPT	
\$50,477.88	

*Submarine ride*

<b>CF<sub>0</sub></b>	-\$1,850,000
<b>C01</b>	\$900,000
<b>F01</b>	1
<b>C02</b>	\$800,000
<b>F02</b>	1
<b>C03</b>	\$750,000
<b>F03</b>	1
I = 14%	
NPV CPT	
\$61,276.34	

12.

*Project I*

<b>CF<sub>0</sub></b>	\$0
<b>C01</b>	\$18,000
<b>F01</b>	3
I = 10%	
NPV CPT	
\$44,763.34	

<b>CF<sub>0</sub></b>	-\$30,000
<b>C01</b>	\$18,000
<b>F01</b>	3
I = 10%	
NPV CPT	
\$14,763.34	

$$PI = \$44,763.34 / \$30,000 = 1.492$$

*Project II*

<b>CF<sub>0</sub></b>	\$0
<b>C01</b>	\$7,500
<b>F01</b>	3
I = 10%	
NPV CPT	
\$18,651.39	

<b>CF<sub>0</sub></b>	-\$12,000
<b>C01</b>	\$7,500
<b>F01</b>	3
I = 10%	
NPV CPT	
\$6,651.39	

$$PI = \$18,651.39 / \$12,000 = 1.554$$



15. a.

<i>CDMA</i>	
<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$11,000,000
<b>F01</b>	1
<b>C02</b>	\$7,500,000
<b>F02</b>	1
<b>C03</b>	\$2,500,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$18,076,634.11

<i>G4</i>	
<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$10,000,000
<b>F01</b>	1
<b>C02</b>	\$25,000,000
<b>F02</b>	1
<b>C03</b>	\$20,000,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$44,778,362.13

<i>Wi-Fi</i>	
<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$18,000,000
<b>F01</b>	1
<b>C02</b>	\$32,000,000
<b>F02</b>	1
<b>C03</b>	\$20,000,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$57,836,213.37

$$PI_{CDMA} = \$18,076,634.11 / \$8,000,000 = 2.26$$

$$PI_{G4} = \$44,778,362.13 / \$12,000,000 = 3.73$$

$$PI_{Wi-Fi} = \$57,836,213.37 / \$20,000,000 = 2.89$$

b.

<i>CDMA</i>	
<b>CF<sub>0</sub></b>	-\$8,000,000
<b>C01</b>	\$11,000,000
<b>F01</b>	1
<b>C02</b>	\$7,500,000
<b>F02</b>	1
<b>C03</b>	\$2,500,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$10,076,634.11

<i>G4</i>	
<b>CF<sub>0</sub></b>	-\$12,000,000
<b>C01</b>	\$10,000,000
<b>F01</b>	1
<b>C02</b>	\$25,000,000
<b>F02</b>	1
<b>C03</b>	\$20,000,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$32,778,362.13

<i>Wi-Fi</i>	
<b>CF<sub>0</sub></b>	-\$20,000,000
<b>C01</b>	\$18,000,000
<b>F01</b>	1
<b>C02</b>	\$32,000,000
<b>F02</b>	1
<b>C03</b>	\$20,000,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$37,836,213.37

16. b.

<i>AZM</i>	
<b>CF<sub>0</sub></b>	-\$450,000
<b>C01</b>	\$320,000
<b>F01</b>	1
<b>C02</b>	\$180,000
<b>F02</b>	1
<b>C03</b>	\$150,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$102,366.64

<i>AZF</i>	
<b>CF<sub>0</sub></b>	-\$800,000
<b>C01</b>	\$350,000
<b>F01</b>	1
<b>C02</b>	\$420,000
<b>F02</b>	1
<b>C03</b>	\$290,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$83,170.55

c.

<i>AZM</i>	
<b>CF<sub>0</sub></b>	-\$450,000
<b>C01</b>	\$320,000
<b>F01</b>	1
<b>C02</b>	\$180,000
<b>F02</b>	1
<b>C03</b>	\$150,000
<b>F03</b>	1

IRR CPT  
24.65%

<i>AZF</i>	
<b>CF<sub>0</sub></b>	-\$800,000
<b>C01</b>	\$350,000
<b>F01</b>	1
<b>C02</b>	\$420,000
<b>F02</b>	1
<b>C03</b>	\$290,000
<b>F03</b>	1

IRR CPT  
15.97%

17. a.

*Project A*

<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$110,000
<b>F01</b>	1
<b>C02</b>	\$110,000
<b>F02</b>	1

I = 12%  
NPV CPT  
\$185,905.61

*Project B*

<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$200,000
<b>F01</b>	1
<b>C02</b>	\$200,000
<b>F02</b>	1

I = 12%  
NPV CPT  
\$338,010.20

*Project C*

<b>CF<sub>0</sub></b>	0
<b>C01</b>	\$120,000
<b>F01</b>	1
<b>C02</b>	\$90,000
<b>F02</b>	1

I = 12%  
NPV CPT  
\$178,890.31

$$PI_A = \$185,905.61 / \$150,000 = 1.24$$

$$PI_B = \$338,010.20 / \$300,000 = 1.13$$

$$PI_C = \$178,890.31 / \$150,000 = 1.19$$

b.

*Project A*

<b>CF<sub>0</sub></b>	-\$150,000
<b>C01</b>	\$110,000
<b>F01</b>	1
<b>C02</b>	\$110,000
<b>F02</b>	1

I = 12%  
NPV CPT  
\$35,905.61

*Project B*

<b>CF<sub>0</sub></b>	-\$300,000
<b>C01</b>	\$200,000
<b>F01</b>	1
<b>C02</b>	\$200,000
<b>F02</b>	1

I = 12%  
NPV CPT  
\$38,010.20

*Project C*

<b>CF<sub>0</sub></b>	-\$150,000
<b>C01</b>	\$120,000
<b>F01</b>	1
<b>C02</b>	\$90,000
<b>F02</b>	1

I = 12%  
NPV CPT  
\$28,890.31

18. b.

*Dry prepeg*

<b>CF<sub>0</sub></b>	-\$1,700,000
<b>C01</b>	\$1,100,000
<b>F01</b>	1
<b>C02</b>	\$900,000
<b>F02</b>	1
<b>C03</b>	\$750,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$607,287.75

*Solvent prepeg*

<b>CF<sub>0</sub></b>	-\$750,000
<b>C01</b>	\$375,000
<b>F01</b>	1
<b>C02</b>	\$600,000
<b>F02</b>	1
<b>C03</b>	\$390,000
<b>F03</b>	1

I = 10%  
NPV CPT  
\$379,789.63

c.

*Dry prepeg*

<b>CF<sub>0</sub></b>	-\$1,700,000
<b>C01</b>	\$1,100,000
<b>F01</b>	1
<b>C02</b>	\$900,000
<b>F02</b>	1
<b>C03</b>	\$750,000
<b>F03</b>	1

IRR CPT  
30.90%

*Solvent prepeg*

<b>CF<sub>0</sub></b>	-\$750,000
<b>C01</b>	\$375,000
<b>F01</b>	1
<b>C02</b>	\$600,000
<b>F02</b>	1
<b>C03</b>	\$390,000
<b>F03</b>	1

IRR CPT  
36.51%

d.

<b>CF<sub>0</sub></b>	-\$950,000
<b>C01</b>	\$725,000
<b>F01</b>	1
<b>C02</b>	\$300,000
<b>F02</b>	1
<b>C03</b>	\$360,000
<b>F03</b>	1
<b>IRR CPT</b>	25.52%

19. b.

<i>NP-30</i>	
<b>CF<sub>0</sub></b>	-\$550,000
<b>C01</b>	\$185,000
<b>F01</b>	5
<b>C02</b>	
<b>F02</b>	
<b>C03</b>	
<b>F03</b>	
<b>C04</b>	
<b>F04</b>	
<b>C05</b>	
<b>F05</b>	
<b>IRR CPT</b>	20.27%

<i>NX-20</i>	
<b>CF<sub>0</sub></b>	-\$350,000
<b>C01</b>	\$100,000
<b>F01</b>	1
<b>C02</b>	\$110,000
<b>F02</b>	1
<b>C03</b>	\$121,000
<b>F03</b>	1
<b>C04</b>	\$133,100
<b>F04</b>	1
<b>C05</b>	\$146,410
<b>F05</b>	1
<b>IRR CPT</b>	20.34%

c.

<i>NP-30</i>	
<b>CF<sub>0</sub></b>	-\$550,000
<b>C01</b>	\$185,000
<b>F01</b>	5
<b>C02</b>	
<b>F02</b>	
<b>C03</b>	
<b>F03</b>	
<b>C04</b>	
<b>F04</b>	
<b>C05</b>	
<b>F05</b>	
<b>I = 15%</b>	
<b>NPV CPT</b>	\$620,148.69

<i>NX-20</i>	
<b>CF<sub>0</sub></b>	-\$350,000
<b>C01</b>	\$100,000
<b>F01</b>	1
<b>C02</b>	\$110,000
<b>F02</b>	1
<b>C03</b>	\$121,000
<b>F03</b>	1
<b>C04</b>	\$133,100
<b>F04</b>	1
<b>C05</b>	\$146,410
<b>F05</b>	1
<b>I = 15%</b>	
<b>NPV CPT</b>	\$398,583.79

$$PI_{NP-30} = \$620,148.69 / \$550,000 = 1.128$$

$$PI_{NX-20} = \$398,583.79 / \$350,000 = 1.139$$



d.

<i>NP-30</i>		<i>NX-20</i>	
<b>CF<sub>0</sub></b>	-\$550,000	<b>CF<sub>0</sub></b>	-\$350,000
<b>C01</b>	\$185,000	<b>C01</b>	\$100,000
<b>F01</b>	5	<b>F01</b>	1
<b>C02</b>		<b>C02</b>	\$110,000
<b>F02</b>		<b>F02</b>	1
<b>C03</b>		<b>C03</b>	\$121,000
<b>F03</b>		<b>F03</b>	1
<b>C04</b>		<b>C04</b>	\$133,100
<b>F04</b>		<b>F04</b>	1
<b>C05</b>		<b>C05</b>	\$146,410
<b>F05</b>		<b>F05</b>	1
I = 15%		I = 15%	
NPV CPT		NPV CPT	
\$70,148.66		\$48,583.79	

28.

<b>CF<sub>0</sub></b>	\$20,000
<b>C01</b>	-\$26,000
<b>F01</b>	1
<b>C02</b>	\$13,000
<b>F02</b>	1
IRR CPT	
ERROR 7	