CHAPTER 8
INTEREST RATES AND BOND VALUATION

Answers to Concept Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.

2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.

3. No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?

4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.

5. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly at par.

6. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provides the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.

7. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.

8. Treasury bonds have no credit risk since it is backed by the U.S. government, so a rating is not necessary. Junk bonds often are not rated because there would be no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).

9. The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.

10. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

11. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."

12. Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.

13. When the bonds are initially issued, the coupon rate is set at auction so that the bonds sell at par value. The wide range of coupon rates shows the interest rate when each bond was issued. Notice that interest rates have evidently declined. Why?

14. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.

15. A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.

16. *a*. The bond price is the present value of the cash flows from a bond. The YTM is the interest rate used in valuing the cash flows from a bond.

b. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.

c. Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

17. A long-term bond has more interest rate risk compared to a short-term bond, all else the same. A low coupon bond has more interest rate risk than a high coupon bond, all else the same. When comparing a high coupon, long-term bond to a low coupon, short-term bond, we are unsure which has more interest rate risk. Generally, the maturity of a bond is a more important determinant of the interest rate risk, so the long-term, high coupon bond probably has more interest rate risk. The exception would be if the maturities are close, and the coupon rates are vastly different.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

NOTE: Most problems do not explicitly list a par value for bonds. Even though a bond can have any par value, in general, corporate bonds in the United States will have a par value of $1,000. We will use this par value in all problems unless a different par value is explicitly stated.

Basic

1. The price of a pure discount (zero coupon) bond is the present value of the par value. Remember, even though there are no coupon payments, the periods are semiannual to stay consistent with coupon bond payments. So, the price of the bond for each YTM is:

a. $P = \$1,000/(1 + .05/2)^{30} = \$476.74$

b. $P = \$1,000/(1 + .10/2)^{30} = \$231.38$

c. $P = \$1,000/(1 + .15/2)^{30} = \$114.22$

2. The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes a semiannual coupon. The price of the bond at each YTM will be:

a. $P = \$35(\{1 - [1/(1 + .035)]^{30}\} / .035) + \$1,000[1 / (1 + .035)^{30}]$
$P = \$1,000.00$
When the YTM and the coupon rate are equal, the bond will sell at par.

b. $P = \$35(\{1 - [1/(1 + .045)]^{30}\} / .045) + \$1,000[1 / (1 + .045)^{30}]$
$P = \$837.11$
When the YTM is greater than the coupon rate, the bond will sell at a discount.

c. $P = \$35(\{1 - [1/(1 + .025)]^{30}\} / .025) + \$1,000[1 / (1 + .025)^{30}]$
$P = \$1,209.30$
When the YTM is less than the coupon rate, the bond will sell at a premium.

3. Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:

$P = \$1,050 = \$32(\text{PVIFA}_{R\%,26}) + \$1,000(\text{PVIF}_{R\%,26})$

Since we cannot solve the equation directly for *R*, using a spreadsheet, a financial calculator, or trial and error, we find:

$R = 2.923\%$

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

$\text{YTM} = 2 \times 2.923\% = 5.85\%$

4. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

P = \$1,060 = C(PVIFA$_{3.8\%,23}$) + \$1,000(PVIF$_{3.8\%,23}$)

Solving for the coupon payment, we get:

C = \$41.96

Since this is the semiannual payment, the annual coupon payment is:

2 × \$41.96 = \$83.92

And the coupon rate is the annual coupon payment divided by par value, so:

Coupon rate = \$83.92 / \$1,000 = .0839 or 8.39%

5. The price of any bond is the PV of the interest payment, plus the PV of the par value. The fact that the bond is denominated in euros is irrelevant. Notice this problem assumes an annual coupon. The price of the bond will be:

P = €45({1 – [1/(1 + .039)]^{19} } / .039) + €1,000[1 / (1 + .039)^{19}]
P = €1,079.48

6. Here we are finding the YTM of an annual coupon bond. The fact that the bond is denominated in yen is irrelevant. The bond price equation is:

P = ¥92,000 = ¥2,800(PVIFA$_{R\%,21}$) + ¥100,000(PVIF$_{R\%,21}$)

Since we cannot solve the equation directly for R, using a spreadsheet, a financial calculator, or trial and error, we find:

R = 3.34%

Since the coupon payments are annual, this is the yield to maturity.

7. The approximate relationship between nominal interest rates (R), real interest rates (r), and inflation (h) is:

$R = r + h$

Approximate r = .045 –.021 =.024 or 2.40%

The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$(1 + .045) = (1 + r)(1 + .021)$

Exact $r = [(1 + .045) / (1 + .021)] – 1 = .0235$ or 2.35%

8. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$(1 + R) = (1 + r)(1 + h)$

$R = (1 + .024)(1 + .031) – 1 = .0557$ or 5.57%

9. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$(1 + R) = (1 + r)(1 + h)$

$h = [(1 + .14) / (1 + .10)] – 1 = .0364$ or 3.64%

10. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$(1 + R) = (1 + r)(1 + h)$

$r = [(1 + .125) / (1.053)] – 1 = .0684$ or 6.84%

11. The coupon rate, located in the first column of the quote is 4.750%. The bid price is:

Bid price = 109:11 = 109 11/32 = 109.34375% × $1,000 = $1,093.4375

The previous day's ask price is found by:

Previous day's asked price = Today's asked price – Change = 119 13/32 – (–11/32) = 119 24/32

The previous day's price in dollars was:

Previous day's dollar price = 109.7500% × $1,000 = $1,097.5000

12. This is a premium bond because it sells for more than 100% of face value. The current yield is:

Current yield = Annual coupon payment / Asked price = $43.75/$1,023.7500 = .0427 or 4.27%

The YTM is located under the "Asked yield" column, so the YTM is 4.2306%.

The bid-ask spread is the difference between the bid price and the ask price, so:

Bid-Ask spread = 102:12 – 102:11 = 1/32

13. Zero coupon bonds are priced with semiannual compounding to correspond with coupon bonds. The price of the bond when purchased was:

$P_0 = \$1,000 / (1 + .035)^{50}$
$P_0 = \$179.05$

And the price at the end of one year is:

$P_0 = \$1,000 / (1 + .035)^{48}$
$P_0 = \$191.81$

So, the implied interest, which will be taxable as interest income, is:

Implied interest = \$191.81 – 179.05
Implied interest = \$12.75

Intermediate

14. Here we are finding the YTM of semiannual coupon bonds for various maturity lengths. The bond price equation is:

$P = C(\text{PVIFA}_{R\%,t}) + \$1,000(\text{PVIF}_{R\%,t})$

Miller Corporation bond:

$P_0 = \$40(\text{PVIFA}_{3\%,26}) + \$1,000(\text{PVIF}_{3\%,26}) = \$1,178.77$
$P_1 = \$40(\text{PVIFA}_{3\%,24}) + \$1,000(\text{PVIF}_{3\%,24}) = \$1,169.36$
$P_3 = \$40(\text{PVIFA}_{3\%,20}) + \$1,000(\text{PVIF}_{3\%,20}) = \$1,148.77$
$P_8 = \$40(\text{PVIFA}_{3\%,10}) + \$1,000(\text{PVIF}_{3\%,10}) = \$1,085.30$
$P_{12} = \$40(\text{PVIFA}_{3\%,2}) + \$1,000(\text{PVIF}_{3\%,2}) = \$1,019.13$
$P_{13} = \$1,000$

Modigliani Company bond:

$P_0 = \$30(\text{PVIFA}_{4\%,26}) + \$1,000(\text{PVIF}_{4\%,26}) = \$840.17$
$P_1 = \$30(\text{PVIFA}_{4\%,24}) + \$1,000(\text{PVIF}_{4\%,24}) = \$847.53$
$P_3 = \$30(\text{PVIFA}_{4\%,20}) + \$1,000(\text{PVIF}_{4\%,20}) = \$864.10$
$P_8 = \$30(\text{PVIFA}_{4\%,10}) + \$1,000(\text{PVIF}_{4\%,10}) = \$918.89$
$P_{12} = \$30(\text{PVIFA}_{4\%,2}) + \$1,000(\text{PVIF}_{4\%,2}) = \$981.14$
$P_{13} = \$1,000$

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

15. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 7 percent. If the YTM suddenly rises to 9 percent:

P_{Laurel} = \$35($PVIFA_{4.5\%,4}$) + \$1,000($PVIF_{4.5\%,4}$) = \$964.12

P_{Hardy} = \$35($PVIFA_{4.5\%,30}$) + \$1,000($PVIF_{4.5\%,30}$) = \$837.11

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_{Laurel}\%$ = (\$964.12 – 1,000) / \$1,000 = –0.0359 or –3.59%

$\Delta P_{Hardy}\%$ = (\$837.11 – 1,000) / \$1,000 = –0.1629 or –16.29%

If the YTM suddenly falls to 5 percent:

P_{Laurel} = \$35($PVIFA_{2.5\%,4}$) + \$1,000($PVIF_{2.5\%,4}$) = \$1,037.62

P_{Hardy} = \$35($PVIFA_{2.5\%,30}$) + \$1,000($PVIF_{2.5\%,30}$) = \$1,209.30

$\Delta P_{Laurel}\%$ = (\$1,037.62 – 1,000) / \$1,000 = +0.0376 or 3.76%

$\Delta P_{Hardy}\%$ = (\$1,209.30 – 1,000) / \$1,000 = +0.2093 or 20.93%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates. Notice also that for the same interest rate change, the gain from a decline in interest rates is larger than the loss from the same magnitude change. For a plain vanilla bond, this is always true.

16. Initially, at a YTM of 10 percent, the prices of the two bonds are:

P_{Faulk} = \$30($PVIFA_{5\%,24}$) + \$1,000($PVIF_{5\%,24}$) = \$724.03

P_{Gonas} = \$70($PVIFA_{5\%,24}$) + \$1,000($PVIF_{5\%,24}$) = \$1,275.97

If the YTM rises from 10 percent to 12 percent:

P_{Faulk} = \$30($PVIFA_{6\%,24}$) + \$1,000($PVIF_{6\%,24}$) = \$623.49

P_{Gonas} = \$70($PVIFA_{6\%,24}$) + \$1,000($PVIF_{6\%,24}$) = \$1,125.50

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_{Faulk}\%$ = (\$623.49 – 724.03) / \$724.03 = –0.1389 or –13.89%
$\Delta P_{Gonas}\%$ = (\$1,125.50 – 1,275.97) / \$1,275.97 = –0.1179 or –11.79%

If the YTM declines from 10 percent to 8 percent:

$P_{Faulk} = \$30(PVIFA_{4\%,24}) + \$1{,}000(PVIF_{4\%,24}) = \$847.53$

$P_{Gonas} = \$70(PVIFA_{4\%,24}) + \$1{,}000(PVIF_{4\%,24}) = \$1{,}457.41$

$\Delta P_{Faulk}\% = (\$847.53 – 724.03) / \$724.03 = +0.1706$ or 17.06%

$\Delta P_{Gonas}\% = (\$1{,}457.41 – 1{,}275.97) / \$1{,}275.97 = +0.1422$ or 14.22%

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

17. The bond price equation for this bond is:

$P_0 = \$1{,}050 = \$31(PVIFA_{R\%,18}) + \$1{,}000(PVIF_{R\%,18})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 2.744\%$

This is the semiannual interest rate, so the YTM is:

$\text{YTM} = 2 \times 2.744\% = 5.49\%$

The current yield is:

Current yield = Annual coupon payment / Price = \$62 / \$1,050 = .0590 or 5.90%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield $= (1 + 0.02744)^2 – 1 = .0556$ or 5.56%

18. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$P = \$1{,}063 = \$35(PVIFA_{R\%,40}) + \$1{,}000(PVIF_{R\%,40})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 3.218\%$

This is the semiannual interest rate, so the YTM is:

$\text{YTM} = 2 \times 3.218\% = 6.44\%$

19. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are two months until the next coupon payment, so four months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = \$68/2 × 4/6 = \$22.67

And we calculate the clean price as:

Clean price = Dirty price – Accrued interest = \$950 – 22.67 = \$927.33

20. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so two months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = \$59/2 × 2/6 = \$9.83

And we calculate the dirty price as:

Dirty price = Clean price + Accrued interest = \$1,053 + 9.83 = \$1,062.83

21. To find the number of years to maturity for the bond, we need to find the price of the bond. Since we already have the coupon rate, we can use the bond price equation, and solve for the number of years to maturity. We are given the current yield of the bond, so we can calculate the price as:

Current yield = .0842 = \$90/P_0
P_0 = \$90/.0842 = \$1,068.88

Now that we have the price of the bond, the bond price equation is:

$P = \$1{,}068.88 = \$90\{[(1 – (1/1.0781)^t\ ] / .0781\} + \$1{,}000/1.0781^t$

We can solve this equation for t as follows:

$\$1{,}068.88\ (1.0781)^t = \$1{,}152.37\ (1.0781)^t – 1{,}152.37 + 1{,}000$
$152.37 = 83.49(1.0781)^t$
$1.8251 = 1.0781^t$
$t = \log 1.8251 / \log 1.0781 = 8.0004 \approx 8$ years

The bond has 8 years to maturity.

22. The bond has 9 years to maturity, so the bond price equation is:

$P = \$1{,}053.12 = \$36.20(\text{PVIFA}_{R\%,18}) + \$1{,}000(\text{PVIF}_{R\%,18})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 3.226\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.226% = 6.45%

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = \$72.40 / \$1,053.12 = .0687 or 6.87%

23. We found the maturity of a bond in Problem 21. However, in this case, the maturity is indeterminate. A bond selling at par can have any length of maturity. In other words, when we solve the bond pricing equation as we did in Problem 21, the number of periods can be any positive number.

24. The price of a zero coupon bond is the PV of the par, so:

a. P_0 = \$1,000/1.035^{50} = \$179.05

b. In one year, the bond will have 24 years to maturity, so the price will be:

P_1 = \$1,000/1.035^{48} = \$191.81

The interest deduction is the price of the bond at the end of the year, minus the price at the beginning of the year, so:

Year 1 interest deduction = \$191.81 – 179.05 = \$12.75

The price of the bond when it has one year left to maturity will be:

P_{24} = \$1,000/1.035^2 = \$933.51

Year 25 interest deduction = \$1,000 – 933.51 = \$66.49

c. Previous IRS regulations required a straight-line calculation of interest. The total interest received by the bondholder is:

Total interest = \$1,000 – 179.05 = \$820.95

The annual interest deduction is simply the total interest divided by the maturity of the bond, so the straight-line deduction is:

Annual interest deduction = \$820.95 / 25 = \$32.84

d. The company will prefer straight-line methods when allowed because the valuable interest deductions occur earlier in the life of the bond.

25. *a*. The coupon bonds have a 6 percent coupon which matches the 6 perent required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell = $45,000,000 / $1,000 = 45,000

The number of zero coupon bonds to sell would be:

Price of zero coupon bonds = $\$1{,}000/1.03^{60}$ = $169.73

Number of zero coupon bonds to sell = $45,000,000 / $169.73 = 265,122

b. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment = 45,000($1,030) = $46,350,000

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes: repayment = 265,122($1,000) = $265,122,140

c. The total coupon payment for the coupon bonds will be the number bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

Coupon bonds: (45,000)($60)(1–.35) = $1,755,000 cash outflow

Note that this is cash outflow since the company is making the interest payment.

For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:

$P_1 = \$1{,}000/1.03^{58} = \$180.07$

The year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part *b*, so:

Year 1 interest deduction per bond = $180.07 – 169.73 = $10.34

The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in year 1 will be:

Cash flows for zeroes in Year 1 = (265,122)($10.34)(.35) = $959,175.00

Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year even though the company did not have a cash flow for the interest expense. This reduces the company's tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.

Challenge

26. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

P: P_0 = \$90($PVIFA_{7\%,10}$) + \$1,000($PVIF_{7\%,10}$) = \$1,140.47

P_1 = \$90($PVIFA_{7\%,9}$) + \$1,000($PVIF_{7\%,9}$) = \$1,130.30

Current yield = \$90 / \$1,140.47 = .0789 or 7.89%

The capital gains yield is:

Capital gains yield = (New price – Original price) / Original price

Capital gains yield = (\$1,130.30 – 1,140.47) / \$1,140.47 = –0.0089 or –0.89%

The current price of Bond D and the price of Bond D in one year is:

D: P_0 = \$50($PVIFA_{7\%,10}$) + \$1,000($PVIF_{7\%,10}$) = \$859.53

P_1 = \$50($PVIFA_{7\%,9}$) + \$1,000($PVIF_{7\%,9}$) = \$869.70

Current yield = \$50 / \$859.53 = 0.0582 or 5.82%

Capital gains yield = (\$869.70 – 859.53) / \$859.53 = 0.0118 or 1.18%

All else held constant, premium bonds pay a high current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 7%, but this return is distributed differently between current income and capital gains.

27. *a*. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

P_0 = \$930 = \$56($PVIFA_{R\%,10}$) + \$1,000($PVIF_{R\%,10}$)

Using a spreadsheet, financial calculator, or trial and error we find:

R = YTM = 6.58%

b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

P_2 = \$56($PVIFA_{5.58\%,8}$) + \$1,000($PVIF_{5.58\%,8}$) = \$1,001.44

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were \$90 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

P_0 = \$930 = \$56($PVIFA_{R\%,2}$) + \$1,001.44($PVIF_{R\%,2}$)

Solving for *R*, we get:

R = HPY = 9.68%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

28. The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we just find the PV of the cash flows. The PV of the cash flows for Bond M is:

P_M = \$800($PVIFA_{4\%,16}$)($PVIF_{4\%,12}$) + \$1,000($PVIFA_{4\%,12}$)($PVIF_{4\%,28}$) + \$30,000($PVIF_{4\%,40}$)
P_M = \$15,200.77

Notice that for the coupon payments of \$800, we found the PVA for the coupon payments, and then discounted the lump sum back to today.

Bond N is a zero coupon bond with a \$30,000 par value; therefore, the price of the bond is the PV of the par, or:

P_N = \$30,000($PVIF_{4\%,40}$) = \$6,248.67

29. In general, this is not likely to happen, although it can (and did). The reason this bond has a negative YTM is that it is a callable U.S. Treasury bond. Market participants know this. Given the high coupon rate of the bond, it is extremely likely to be called, which means the bondholder will not receive all the cash flows promised. A better measure of the return on a callable bond is the yield to call (YTC). The YTC calculation is the basically the same as the YTM calculation, but the number of periods is the number of periods until the call date. If the YTC were calculated on this bond, it would be positive.

30. To find the present value, we need to find the real weekly interest rate. To find the real return, we need to use the effective annual rates in the Fisher equation. So, we find the real EAR is:

$(1 + R) = (1 + r)(1 + h)$
$1 + .069 = (1 + r)(1 + .032)$
$r = .0359$ or 3.59%

Now, to find the weekly interest rate, we need to find the APR. Using the equation for discrete compounding:

EAR = $[1 + (\text{APR} / m)]^m - 1$

We can solve for the APR. Doing so, we get:

APR = $m[(1 + \text{EAR})^{1/m} - 1]$
APR = $52[(1 + .0359)^{1/52} - 1]$
APR = .0352 or 3.52%

So, the weekly interest rate is:

Weekly rate = APR / 52
Weekly rate = .0352 / 52
Weekly rate = .0007 or 0.07%

Now we can find the present value of the cost of the roses. The real cash flows are an ordinary annuity, discounted at the real interest rate. So, the present value of the cost of the roses is:

PVA = $C(\{1 - [1/(1 + r)]^t \} / r)$
PVA = $\$8(\{1 - [1/(1 + .0007)]^{30(52)}\} / .0007)$
PVA = $7,702.30

31. To answer this question, we need to find the monthly interest rate, which is the APR divided by 12. We also must be careful to use the real interest rate. The Fisher equation uses the effective annual rate, so, the real effective annual interest rates, and the monthly interest rates for each account are:

Stock account:
$(1 + R) = (1 + r)(1 + h)$
$1 + .12 = (1 + r)(1 + .04)$
r = .0769 or 7.69%

APR = $m[(1 + \text{EAR})^{1/m} - 1]$
APR = $12[(1 + .0769)^{1/12} - 1]$
APR = .0743 or 7.43%

Monthly rate = APR / 12
Monthly rate = .0743 / 12
Monthly rate = .0062 or 0.62%

Bond account:
$(1 + R) = (1 + r)(1 + h)$
$1 + .07 = (1 + r)(1 + .04)$
r = .0288 or 2.88%

APR = $m[(1 + \text{EAR})^{1/m} - 1]$
APR = $12[(1 + .0288)^{1/12} - 1]$
APR = .0285 or 2.85%

Monthly rate = APR / 12
Monthly rate = .0285 / 12
Monthly rate = .0024 or 0.24%

Now we can find the future value of the retirement account in real terms. The future value of each account will be:

Stock account:
FVA = C {(1 + r)^t – 1] / r}
FVA = \$900{[(1 + .0062)^{360} – 1] / .0062]}
FVA = \$1,196,731.96

Bond account:
FVA = C {(1 + r)^t – 1] / r}
FVA = \$300{[(1 + .0024)^{360} – 1] / .0024]}
FVA = \$170,316.78

The total future value of the retirement account will be the sum of the two accounts, or:

Account value = \$1,196,731.96 + 170,316.78
Account value = \$1,367,048.74

Now we need to find the monthly interest rate in retirement. We can use the same procedure that we used to find the monthly interest rates for the stock and bond accounts, so:

$(1 + R) = (1 + r)(1 + h)$
$1 + .08 = (1 + r)(1 + .04)$
$r = .0385$ or 3.85%

APR = m[(1 + EAR)$^{1/m}$ – 1]
APR = 12[(1 + .0385)$^{1/12}$ – 1]
APR = .0378 or 3.78%

Monthly rate = APR / 12
Monthly rate = .0378 / 12
Monthly rate = .0031 or 0.31%

Now we can find the real monthly withdrawal in retirement. Using the present value of an annuity equation and solving for the payment, we find:

PVA = C({1 – [1/(1 + r)]^t } / r)
\$1,367,048.74 = C({1 – [1/(1 + .0031)]^{300} } / .0031)
C = \$7,050.75

This is the real dollar amount of the monthly withdrawals. The nominal monthly withdrawals will increase by the inflation rate each month. To find the nominal dollar amount of the last withdrawal, we can increase the real dollar withdrawal by the inflation rate. We can increase the real withdrawal by the effective annual inflation rate since we are only interested in the nominal amount of the last withdrawal. So, the last withdrawal in nominal terms will be:

$FV = PV(1 + r)^t$
$FV = \$7,050.75(1 + .04)^{(30 + 25)}$
$FV = \$60,963.34$

32. In this problem, we need to calculate the future value of the annual savings after the five years of operations. The savings are the revenues minus the costs, or:

Savings = Revenue – Costs

Since the annual fee and the number of members are increasing, we need to calculate the effective growth rate for revenues, which is:

Effective growth rate = (1 + .06)(1 + .03) – 1
Effective growth rate = .0918 or 9.18%

The revenue for the current year is the number of members times the annual fee, or:

Current revenue = 600($500)
Current revenue = $300,000

The revenue will grow at 9.18 percent, and the costs will grow at 2 percent, so the savings each year for the next five years will be:

Year	*Revenue*	*Costs*	*Savings*
1	$327,540.00	$127,500.00	$200,040.00
2	357,608.17	130,050.00	227,558.17
3	390,436.60	132,651.00	257,785.60
4	426,278.68	135,304.02	290,974.66
5	465,411.07	138,010.10	327,400.96

Now we can find the value of each year's savings using the future value of a lump sum equation, so:

$FV = PV(1 + r)^t$

Year		*Future Value*
1	$\$200,040.00(1 + .09)^4 =$	$282,372.79
2	$\$227,558.17(1 + .09)^3 =$	294,694.43
3	$\$257,785.60(1 + .09)^2 =$	306,275.07
4	$\$290,974.66(1 + .09)^1 =$	317,162.38
5		327,400.96
Total future value of savings =		$1,527,905.64

He will spend \$500,000 on a luxury boat, so the value of his account will be:

Value of account = \$1,527,905.64 – 500,000
Value of account = \$1,027,905.64

Now we can use the present value of an annuity equation to find the payment. Doing so, we find:

$\text{PVA} = C(\{1 - [1/(1 + r)]^t \} / r)$
$\$1,027,905.64 = C(\{1 - [1/(1 + .09)]^{25} \} / .09)$
$C = \$104,647.22$

Calculator Solutions

1.
a.

Enter	30	2.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$476.74		

b.

Enter	30	5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$231.38		

c.

Enter	30	7.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$114.22		

2.
a.

Enter	30	3.5%		$35	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,000.00		

b.

Enter	30	4.5%		$35	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$837.11		

c.

Enter	30	2.5%		$35	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,209.30		

3.

Enter	26		±$1,050	$32	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		2.923%			

2.923% × 2 = 5.85%

4.

Enter	23	3.8%	±$1,060		$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$41.96	

$41.96 × 2 = $83.92
$83.92 / $1,000 = 8.39%

5.

Enter	19	3.90%		€45	€1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			€1,079.48		

6.

Enter	21		±¥92,000	¥2,800	¥100,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		3.34%			

13.

P_0

Enter	50	3.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$179.05		

P_1

Enter	48	3.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$191.81		

$191.81 – 179.05 = $12.75

14. Miller Corporation

P_0

Enter	26	3%		$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,178.77		

P_1

Enter	24	3%		$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,169.36		

P_3

Enter	20	3%		$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,148.77		

P_8

Enter	10	3%		$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,085.30		

P_{12}

Enter	2	3%		$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,019.13		

Modigliani Company

P_0

	N	I/Y	PV	PMT	FV
Enter	26	4%		$30	$1,000
Solve for			$840.17		

P_1

	N	I/Y	PV	PMT	FV
Enter	24	4%		$30	$1,000
Solve for			$847.53		

P_3

	N	I/Y	PV	PMT	FV
Enter	20	4%		$30	$1,000
Solve for			$864.10		

P_8

	N	I/Y	PV	PMT	FV
Enter	10	4%		$30	$1,000
Solve for			$918.89		

P_{12}

	N	I/Y	PV	PMT	FV
Enter	2	4%		$30	$1,000
Solve for			$981.14		

15. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 7 percent. If the YTM suddenly rises to 9 percent:

P_{Laurel}

	N	I/Y	PV	PMT	FV
Enter	4	4.5%		$35	$1,000
Solve for			$964.12		

$\Delta P_{Laurel}\% = (\$964.12 – 1,000) / \$1,000 = –3.59\%$

P_{Hardy}

	N	I/Y	PV	PMT	FV
Enter	30	4.5%		$35	$1,000
Solve for			$837.11		

$\Delta P_{Hardy}\% = (\$837.11 – 1,000) / \$1,000 = –16.29\%$

If the YTM suddenly falls to 5 percent:

P_{Laurel}

	N	I/Y	PV	PMT	FV
Enter	4	2.5%		$35	$1,000
Solve for			$1,037.62		

$\Delta P_{Laurel}\% = (\$1,037.62 – 1,000) / \$1,000 = + 3.76\%$

P_{Hardy}

Enter	30	2.5%		$35	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,209.93		

$\Delta P_{Hardy}\% = (\$1,209.93 – 1,000) / \$1,000 = + 20.93\%$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

16. Initially, at a YTM of 10 percent, the prices of the two bonds are:

P_{Faulk}

Enter	24	5%		$30	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$724.03		

P_{Gonas}

Enter	24	5%		$70	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,275.97		

If the YTM rises from 10 percent to 12 percent:

P_{Faulk}

Enter	24	6%		$30	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$623.49		

$\Delta P_{Faulk}\% = (\$623.49 – 724.03) / \$724.03 = –13.89\%$

P_{Gonas}

Enter	24	6%		$70	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,125.50		

$\Delta P_{Gonas}\% = (\$1,125.50 – 1,275.97) / \$1,275.97 = –11.79\%$

If the YTM declines from 10 percent to 8 percent:

P_{Faulk}

Enter	24	4%		$30	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$847.53		

$\Delta P_{Faulk}\% = (\$847.53 – 724.03) / \$724.03 = +17.06\%$

P_{Gonas}

Enter	24	4%		$70	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,457.41		

$\Delta P_{Gonas}\% = (\$1,457.41 – 1,275.97) / \$1,275.97 = +14.22\%$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

17.

	N	I/Y	PV	PMT	FV
Enter	18		±$1,050	$31	$1,000
Solve for		2.744%			

YTM = 2.744% × 2 = 5.49%

18. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

	N	I/Y	PV	PMT	FV
Enter	40		±$1,063	$35	$1,000
Solve for		3.218%			

3.218% × 2 = 6.44%

21. Current yield = .0842 = $90/$P_0$; P_0 = $1,068.88

	N	I/Y	PV	PMT	FV
Enter		7.81%	±$1,068.88	$90	$1,000
Solve for	8.0004				

8 years

22.

	N	I/Y	PV	PMT	FV
Enter	18		±$1,053.12	$36.20	$1,000
Solve for		3.226%			

3.226% × 2 = 6.45%

24.

a. P_o

	N	I/Y	PV	PMT	FV
Enter	50	7%/2			$1,000
Solve for			$179.05		

b. P_1

	N	I/Y	PV	PMT	FV
Enter	48	7%/2			$1,000
Solve for			$191.81		

year 1 interest deduction = $191.81 – 179.05 = $12.75

P_{24}

	N	I/Y	PV	PMT	FV
Enter	2	7%/2%			$1,000
Solve for			$933.51		

year 25 interest deduction = $1,000 – 933.51 = $66.49

c. Total interest = $1,000 – 179.05 = $820.95
Annual interest deduction = $820.95 / 25 = $32.84

d. The company will prefer straight-line method when allowed because the valuable interest deductions occur earlier in the life of the bond.

25. *a*. The coupon bonds have a 6% coupon rate, which matches the 6% required return, so they will sell at par; # of bonds = $45,000,000/$1,000 = 45,000.

For the zeroes:

Enter	60	6%/2			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$169.73		

$45,000,000/$169.73 = 265,122 will be issued.

b. Coupon bonds: repayment = 45,000($1,030) = $46,350,000
Zeroes: repayment = 265,122($1,000) = $265,122,140

c. Coupon bonds: (45,000)($60)(1 –.35) = $1,755,000 cash outflow
Zeroes:

Enter	58	6%/2			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$180.07		

year 1 interest deduction = $180.07 – 169.73 = $10.34
(265,122)($10.34)(.35) = $959,175.00 cash inflow
During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.

26.
Bond P
P_0

Enter	10	7%		$90	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,140.47		

P_1

Enter	9	7%		$90	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,130.30		

Current yield = $90 / $1,140.47 = 7.89%
Capital gains yield = ($1,130.30 – 1,140.47) / $1,140.47 = –0.89%

Bond D
P_0

Enter	10	7%		$50	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$859.53		

P_1

Enter	9	7%		$50	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$869.70		

Current yield = $50 / $859.53 = 5.82%
Capital gains yield = ($869.70 – 859.53) / $859.53 = +1.18%

All else held constant, premium bonds pay a higher current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 7%, but this return is distributed differently between current income and capital gains.

27.

a.

Enter	10		±$930	$56	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		6.58%			

This is the rate of return you expect to earn on your investment when you purchase the bond.

b.

Enter	8	5.58%		$56	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,001.44		

The HPY is:

Enter	2		±$930	$56	$1,001.44
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		9.68%			

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

28.

P_M

CFo	$0
C01	$0
F01	12
C02	$800
F02	16
C03	$1,000
F03	11
C04	$31,000
F04	1

I = 4%
NPV CPT
$15,200.77

P_N

Enter	40	4%			$30,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$6,248.67		

31.

Real return for stock account: 1 + .12 = (1 + r)(1 + .04); r = 7.6923%

Enter		7.6923%	12
	NOM	**EFF**	**C/Y**
Solve for	7.4337%		

Real return for bond account: 1 + .07 = (1 + r)(1 + .04); r = 2.8846%

Enter		2.8846%	12
	NOM	**EFF**	**C/Y**
Solve for	2.8472%		

Real return post-retirement: 1 + .08 = (1 + r)(1 + .04); r = 3.8462%

Enter		3.8462%	12
	NOM	**EFF**	**C/Y**
Solve for	3.7800%		

Stock portfolio value:

Enter	12 × 30	7.4337% / 12		$800	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,196,731.96

Bond portfolio value:

Enter	12 × 30	2.8472% / 12		$400	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$170,316.78

Retirement value = $1,196,731.96 + 170,316.78 = $1,367,048.74

Retirement withdrawal:

Enter	25 × 12	3.7800% / 12	$1,367,048.74		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$7,050.75	

The last withdrawal in real terms is:

Enter	30 + 25	4%	$7,050.75		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$60,963.34

32.
Future value of savings:
Year 1:

Enter	4	9%	$200,040		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$282,372.79

Year 2:

Enter	3	9%	$227,558.17		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$294,694.43

Year 3:

Enter	2	9%	$257,785.60		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$306,275.07

Year 4:

Enter	1	9%	$290,974.66		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$317,162.38

Future value = $282,372.79 + 294,694.43 + 306,275.07 + 317,162.38 + 327,400.96
Future value = $1,527,905.64

He will spend $500,000 on a luxury boat, so the value of his account will be:

Value of account = $1,527,905.64 – 500,000
Value of account = $1,027,905.64

Enter	25	9%	$1,027,905.64		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$104,647.22	