Asset Pricing Part 1 – Chapt. 11 in RWJJ

**Here, we are dealing with probabilities of future returns – not historical data, so we will calculate Expected Return and Standard Deviation differently than we did in chapter 10.**

**Expected Return** - Probability weighted average of possible outcomes

E(Ri) = Expected Return for asset i

E(Ri) =  Pt Rit

Pt = Probability of outcome t

Rit = Return for asset i if outcome t occurs

Sum of probabilities = 1.0

**Variance** - Average value of squared deviations from the expected return.

σ2 =  [Pt (Rit- E(Ri))2] Note that we use probabilities and don’t divide by T or T-1

**Standard Deviation** - Square root of the variance = σ This is our definition of risk

**Example**:

Returns and their Probabilities

-10% 0% 10% 20% 30%

Stock

A 1.00

B .25 .50 .25

C .10 .20 .40 .20 .10

**Expected Return**:

E(RA) = 1 (10%) = 10%

E(RB) = .25 (0%) + .50 (10%) + .25 (20%)

0 + 5% + 5% = 10%

E(RC) = .10 (-10%) + .20 (0%) + .40 (10%) + .20 (20%) + .10 (30%)

= -1% + 0% + 4% + 4% + 3% = 10%

E(RA) = E(RB) = E(RC)

**Variance**: Note that it is easier to use decimals here.

σ2A = 1 (.1 - .1)2 = 1(0) = 0

σ2B = .25 (0 - .1)2 + .5 (.1 - .1)2 + .25 (.2 - .1)2

= .25 (.01) + .5 (0) + .25 (.01)

= .0025 + 0 + .0025 = .005

σ2C = .1 (-.1 - .1)2 + .2(0 - .1)2 + .4 (.1 - .1)2 + .2 (.2 - .1)2 + .1 (.3 - .1)2

= .1 (.04) + .2 (.01) + .4 (0) + .2 (.01) + .1 (.04)

= .004 + .002 + 0 + .002 + .004 = .012

σ2A < σ2B < σ2C

**Standard Deviation**:

σA =  = 0

σB =  = .071 = 7.1%

σC =  = .11 = 11%

σA < σB < σC

By our definition of risk, C is most risky and A is least risky.

**Risk Averse** - Someone who wants more expected return and less standard deviation

**Risk Loving** - Someone who wants more expected return and more standard deviation

**Risk Neutral** - Someone who wants more expected return and doesn’t care about standard deviation

Most investors are Risk Averse – Modern Portfolio Theory works under the assumption that all are. However, while all investors are risk averse, some are more risk averse than others.

If no one wants C, and everyone wants A, what happens?

The price of A is bid up and the price of C is bid down. This increases the expected return of C, and decreases the expected return of A.

**Risk-Free Return** - The return you get from investing in an asset with a standard deviation of zero.

In our example, E(RA) = Risk-Free rate of return = Rf

**Risk Premium** - The expected return in excess of the risk-free return that an investor requires as compensation for bearing risk.

This is because risk-averse investors are only willing to take on more risk (standard deviation) if they are compensated with a higher expected return.

(Rm – Rf) is a special risk premium. It is the Market Risk Premium. The expected return in excess of the risk-free rate that an investor demands as compensation for bearing the risk of investing in the market.

**Covariance** - The extent to which the returns on two assets tend to move together.

CovA,B =  P­t[(RAt - E(RA)) (RBt - E(RB))]

Note the similarity to Variance: Covariance and variance are the same thing. Variance involves only one asset while covariance involves two assets. Notice that the covariance between any asset and itself is its variance:

CovA,A = VarA =  P­t[(RAt - E(RA)) (RAt - E(RA))] =  [Pt (RAt- E(RA))2]

**Covariance Example**:

Economy Prob. A B

Normal .4 11% 9%

Strong .2 20 4

Weak .2 7 17

Inflation .1 12 -5

Recession .1 -10 7

E(RA) = .4(11%) + .2(20%) + .2(7%) + .1(12%) + .1(-10%) = 10%

E(RB) = .4(9%) + .2(4%) + .2(17%) + .1(-5%) + .1(7%) = 8%

CovA,B  = .4 (.11 - .10) (.09 - .08) + .2 (.20 - .10) (.04 - .08) +

.2 (.07 - .10) (.17 - .08) + .1 (.12 - .10) (-.05 - .08) + .1 (-.10 - .10) (.07 - .08)

= -.00136 As with variance, it is easier to use decimals with covariance

**Negative Covariance** = The two assets tend to move in opposite directions

When one is above (below) its mean, the other is more likely to

be below (above) its mean

**Positive Covariance** = The two assets tend to move in the same direction

When one is above (below) its mean, the other is more likely to

be above (below) its mean

**Zero Covariance** = Random movement – Or one asset doesn’t move at all

Correlation Coefficient = ρ = Cov(A,B) -1 < ρ < +1

σA σB

For assets A and B in our example above, ρ = -.28335

Most stocks have a positive covariance with each other, but they don’t move perfectly together. So for most stocks: 0 < ρ < 1

Definition: Portfolio - Two or more securities looked at as a group (technically, it can be one asset)

A portfolio is a team and you are the coach. A team is a collection of individual players. A portfolio is a collection of individual assets. As the coach, your goal is for the team to do well. You don’t just look at the performance of the individual players – but how they contribute to the performance of the team. In the same way, we want to see how individual assets contribute to the expected return of the portfolio and the standard deviation of the portfolio.

**Portfolio Expected Return**

The expected return of any portfolio is a weighted average of the expected returns of the assets in the portfolio, weighted by the proportion of our money that we invest in each asset. Thus, the weights must always add up to 1.0

E(RP) = w1E(R1) + w2E(R2) + w3E(R3) + … where w = the weight on the asset

**Example**:

Stock A: Expected Return = 10% Stock B: Expected Return = 15%

Portfolio AB - 50% in A and 50% in B .5 (10%) + .5 (15%) = 12.5%

75% in A and 25% in B .75 (10%) + .25 (15%) = 11.25%

25% in A and 75% in B .25 (10%) + .75 (15%) = 13.75%

**Portfolio Standard Deviation**

For any two-asset portfolio, if the two assets do not move exactly together (if ρ < 1), the variance (standard deviation) of the portfolio will always be less than the weighted average of the variances (standard deviations) of the individual assets.

**Example**:

Stock X - Expected Return = 10%

Standard Deviation = 5%

Stock Y - Expected Return = 10%

Standard Deviation = 7%

Portfolio XY (with 50% in each stock) - Expected Return = 10%

Standard Deviation will be < 6% if ρ < 1.0

The lower the covariance, the lower will be the portfolio standard deviation. If we have a perfect negative correlation (ρ = -1), the portfolio standard deviation will equal zero for some combination of X and Y.

Suppose the standard deviation of Portfolio XY is 4%

Do risk-averse investors want Stock X, Stock Y, or Portfolio XY?

Answer: Portfolio XY, because it has the same expected return as either X or Y by themselves, but with a lower standard deviation than either of them.

So a portfolio’s expected return is a weighted average of the expected returns of the stocks that are in it, but a portfolio’s standard deviation generally gets lower as you add more stocks that don’t perfectly covary with each other.

Note the figure in your text which shows how standard deviation decreases as the number of securities in a portfolio increases.

**Diversification** - A strategy designed to reduce risk (standard deviation) by spreading the risk across many investments. Don’t put all your eggs in one basket.

Diversification reduces the standard deviation of a portfolio.

There are two types of risk we are concerned with:

**Unique risk** = Unsystematic Risk = Diversifiable Risk. Risk factors affecting only that stock.

**Market risk** = Systematic Risk = Non-diversifiable Risk. Economy-wide sources of risk that affect the overall market and thus all stocks.

The addition of the two is the risk of an individual stock. You can get rid of the first (Unique Risk) through diversification, but you can never totally get rid of the second (Market Risk).

**Examples of Unique Risk**

1. I hold stock in one clothing store. It has a fire and loses its entire inventory.

I hold stock in 50 different clothing stores. Will this happen to all of them?

2. I hold stock in one energy trading company in Texas. It is discovered that the company was not making the money they claimed to be making due to accounting fraud. Within days of the discovery, its stock is worthless.

I hold stock in 10 different companies. Will this happen to all of them?

These were examples of unique risk - things that can happen to any firm, but won’t happen to them all at the same time.

**Market risk**: things that can and do affect all firms at the same time:

Interest Rate Changes

Recession

Changes in Technology

Political Changes

Inflation

Not each of these things affects every stock in the same way, but they do affect the entire economy – every stock feels the effect.

Diversification will eliminate Unique Risk but not Market Risk.

If investors are risk-averse, there is no reason for them to hold individual stocks - just diversified portfolios because they want to get rid of the unique risk.

Thus, if no one holds individual stocks, the risk of an individual security held as part of a portfolio depends on how it affects the volatility of the portfolio, not the volatility of the individual stock.

The risk of a security held by itself is its variance (or standard deviation).

The risk of a security held as part of a portfolio is how it affects the *portfolio* variance (or standard deviation).

Investors should only be concerned with market risk, because they can eliminate unique risk through diversification.

Market Risk - How much an individual stock reacts to economy-wide events.

Some stocks have more market risk than others:

**Example** of a portfolio return and portfolio risk using our prior example:

Suppose we invest 50% of our money in A and 50% in B.

Economy Prob. A B AB

Normal .4 11% 9% 10%

Strong .2 20 4 12

Weak .2 7 17 12

Inflation .1 12 -5 3.5

Recession .1 -10 7 -1.5

E(RA) = .4(11%) + .2(20%) + .2(7%) + .1(12%) + .1(-10%) = 10%

E(RB) = .4(9%) + .2(4%) + .2(17%) + .1(-5%) + .1(7%) = 8%

E(RAB) = Expected Return on Portfolio AB:

.4 (10%) + .2 (12%)+ .2 (12%) + .1 (3.5%) + .1 (-1.5%) = 9.0% = E(RAB)

Or, we can calculate it as: .5 (10%) + .5 (8%) = 9.0%

Note that the expected return on the portfolio is a weighted average of the expected returns on the individual stocks. The correlation (or covariance) between the stocks does not have any effect on the portfolio’s expected return.

**Portfolio Variance** = wA2 σ A2 + wB2 σB2 + 2wAwB Cov(A,B)

where wA = proportion of your money invested in A, and wB = proportion in B.

Note that your textbook uses X for the proportion, but I will use w (weight).

First we must calculate the covariance between A and B and their individual variances.

CovA,B  = .4 (.11 - .10) (.09 - .08) + .2 (.20 - .10) (.04 - .08) +

.2 (.07 - .10) (.17 - .08) + .1 (.12 - .10) (-.05 - .08) + .1 (-.10 - .10) (.07 - .08)

= -.00136

σ2A  = .4(.11 - .10)2 + .2(.20 - .10)2 + .2(.07 - .10)2 + .1 (.12 - .10)2 + .1(-.10 - .10)2

= .00626

σ2B = .4(.09 - .08)2 + .2(.04 - .08)2 + .2(.17 - .08)2 + .1(-.05 - .08)2 + .1(.07 - .08)2

= .00368

σ2AB with 50% in A and 50% in B:

= (.5)2 (.00626) + (.5)2 (.00368) + (2) (.5) (.5) (-.00136)

= .001805

σ2AB < σ2A or σ2B

.001805 < .00626 or .00368

For Standard Deviation we get: σA =  = 7.9%

σB =  = 6.1%

σAB =  = 4.25%

If this is the entire economy - just these two stocks:

# Market Risk = 4.25% (standard deviation) - risk that can’t be diversified away

Unique Risk = 3.65% for A and 1.85% for B – risk that can be diversified away

Total Risk = 7.9% for A and 6.1% for B – their individual standard deviations

You can see this more easily with the Matrix Approach

Variances are the diagonal terms

Covariances are the off-diagonal terms

Note that while the expected return of a portfolio is simply a weighted average of the expected returns of the individual stocks (weighted by the proportion of your money that you invest in each stock), the standard deviation of a portfolio is **not** simply a weighted average of the standard deviation of the individual stocks. A lower covariance (or correlation) between the stocks means a lower standard deviation for the portfolio.