***CHAPTER 12***

**AN ALTERNATIVE VIEW OF RISK AND RETURN: *THE ARBITRAGE PRICING THEORY***

**Answers to Concept Questions**

**1.** Systematic risk is risk that cannot be diversified away through formation of a portfolio. Generally, systematic risk factors are those factors that affect a large number of firms in the market, however, those factors will not necessarily affect all firms equally. Unsystematic risk is the type of risk that can be diversified away through portfolio formation. Unsystematic risk factors are specific to the firm or industry. Surprises in these factors will affect the returns of the firm in which you are interested, but they will have no effect on the returns of firms in a different industry and perhaps little effect on other firms in the same industry.

**2.** Any return can be explained with a large enough number of systematic risk factors. However, for a factor model to be useful as a practical matter, the number of factors that explain the returns on an asset must be relatively limited.

**3.** The market risk premium and inflation rates are probably good choices. The price of wheat, while a risk factor for Ultra Bread, is not a market risk factor and will not likely be priced as a risk factor common to all stocks. In this case, wheat would be a firm specific risk factor, not a market risk factor. A better model would employ macroeconomic risk factors such as interest rates, GDP, energy prices, and industrial production, among others.

**4.** *a.* Real GNP was higher than anticipated. Since returns are positively related to the level of GNP, returns should rise based on this factor.

*b.* Inflation was exactly the amount anticipated. Since there was no surprise in this announcement, it will not affect Lewis-Striden returns.

*c.* Interest rates are lower than anticipated. Since returns are negatively related to interest rates, the lower than expected rate is good news. Returns should rise due to interest rates.

*d.* The president’s death is bad news. Although the president was expected to retire, his retirement would not be effective for six months. During that period, he would still contribute to the firm. His untimely death means that those contributions will not be made. Since he was generally considered an asset to the firm, his death will cause returns to fall. However, since his departure was expected soon, the drop might not be very large.

*e.* The poor research results are also bad news. Since Lewis-Striden must continue to test the drug, it will not go into production as early as expected. The delay will affect expected future earnings, and thus it will dampen returns now.

*f.* The research breakthrough is positive news for Lewis-Striden. Since it was unexpected, it will cause returns to rise.

*g.* The competitor’s announcement is also unexpected, but it is not a welcome surprise. This announcement will lower the returns on Lewis-Striden.

The systematic factors in the list are real GNP, inflation, and interest rates. The unsystematic risk factors are the loss of the president’s ability to contribute to the firm, the research results, and the competitor.

**5.** The main difference is that the market model assumes that only one factor, usually a stock market aggregate, is enough to explain stock returns, while a *k*-factor model relies on *k* factors to explain returns.

**6.** The fact that APT does not give any guidance about the factors that influence stock returns is a commonly-cited criticism. However, in choosing factors, we should choose factors that have an economically valid reason for potentially affecting stock returns. For example, a smaller company has more risk than a larger company. Therefore, the size of a company can affect the returns of the company stock.

**7.** Assuming the market portfolio is properly scaled, it can be shown that the one-factor model is identical to the CAPM.

**8.** It is the weighted average of expected returns plus the weighted average of each security's beta times a factor *F* plus the weighted average of the unsystematic risks of the individual securities.

**9.** Choosing variables because they have been shown to be related to returns is data mining. The relation found between some attribute and returns can be accidental, thus overstated. For example, the occurrence of sunburns and ice cream consumption are related; however, sunburns do not necessarily cause ice cream consumption, or vice versa. For a factor to truly be related to asset returns, there should be sound economic reasoning for the relationship, not just a statistical one.

**10.** Using a benchmark composed of British stocks is wrong because the stocks included likely face a different set of risks and opportunities than those in a U.S. growth stock fund.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** Since we have the expected return of the stock, the revised expected return can be determined using the innovation, or surprise, in the risk factors. So, the revised expected return is:

*R* = .111 + 1.25(.024 – .028) – .47(.027 – .026)

*R* = .1055, or 10.55%

**2.** *a.* If *m* is the systematic risk portion of return, then:

*m* = βGDPΔGDP + βInflationΔInflation + βrΔInterest rates

*m* = .0000615($23,187 – 22,432) – .90(.0260 – .0230) – .32(.0310 – .0280)

*m* = .0428, or 4.28%

*b.* The unsystematic return is the return that occurs because of a firm specific factor such as the bad news about the company. So, the unsystematic return of the stock is –1.25 percent. The total return is the expected return, plus the two components of unexpected return: the systematic risk portion of return and the unsystematic portion. So, the total return of the stock is:

*R* =  + *m* + ε

*R* = .1150 + .0428 – .0125

*R* = .1453, or 14.53%

**3.** *a.* If *m* is the systematic risk portion of return, then:

*m* = βGNPΔ%GNP + β*r*ΔInterest rates

*m* = 1.35(.024 – .021) – .87(.036 – .033)

*m* = .0014, or .14%

*b.* The unsystematic return is the return that occurs because of a firm specific factor such as the increase in market share. If ε is the unsystematic risk portion of the return, then:

ε = .75(.15 – .12)

ε = .0225, or 2.25%

*c.* The total return is the expected return, plus the two components of unexpected return: the systematic risk portion of return and the unsystematic portion. So, the total return of the stock is:

*R* =  + *m* + ε

*R* = .122 + .0014 + .0225

*R* = .1459, or 14.59%

**4.** The beta for a particular risk factor in a portfolio is the weighted average of the betas of the assets. This is true whether the betas are from a single factor model or a multifactor model. So, the betas of the portfolio are:

*F*1 = .20(1.45) + .20(.87) + .60(.76)

*F*1 = .92

*F*2 = .20(.85) + .20(1.35) + .60(–.26)

*F*2 = .28

*F*3 = .20(.10) + .20(–.30) + .60(1.19)

*F*3 = .67

So, the expression for the return of the portfolio is:

*Ri* = 3.2% + .92*F*1 + .28*F*2 – .67*F*3

Which means the return of the portfolio is:

*Ri* = 3.2% + .92(5.30%) + .28(4.10%) – .67(5.90%)

*Ri* = 5.26%

*Intermediate*

**5.** We can express the multifactor model for each portfolio as:

E(*RP*) = *RF* + 1*F*1 + 2*F*2

where *F*1 and *F*2 are the respective risk premiums for each factor. Expressing the return equation for each portfolio, we get:

16% = 4% + .85*F*1 + 1.15*F*2

12% = 4% + 1.45*F*1 – .25*F*2

We can solve the system of two equations with two unknowns. Multiplying each equation by the respective *F*2 factor for the other equation, we get:

4.00% = 1.0% + .2125*F*1 + .2875*F*2

13.8% = 4.6% + 1.6675*F*1 – .2875*F*2

Summing the equations and solving for *F*1 gives us:

17.8% = 5.6% + 1.88*F*1

*F*1 = 6.49%

And now, using the equation for Portfolio A, we can solve for *F*2, which is:

12% = 4% + 1.45(6.49%) – .25*F*2

*F*2 = 5.64%

**6.** *a.* The market model is specified by:

*R* = + β(*R*M – ) + ε

so applying that to each stock:

Stock A:

*R*A = + βA(*RM* – ) + εA

*R*A = 10.5% + 1.2(*RM* – 14.2%) + εA

Stock B:

*R*B = + βB(*RM* – ) + εB

*R*B = 13.0% + .98(*RM* – 14.2%) + εB

Stock C:

*R*C = + βC(*RM* – ) + εC

*R*C = 15.7% + 1.37(*RM* – 14.2%) + εC

*b.* Since we don't have the actual market return or unsystematic risk, we will get a formula with those values as unknowns:

*RP* = .30*R*A + .45*R*B + .25*R*C

*RP* = .30[10.5% + 1.2(*RM* – 14.2%) + εA] + .45[13.0% + .98(*RM* – 14.2%) + εB]

+ .25[15.7% + 1.37(*RM* – 14.2%) + εC]

*RP* = .30(10.5%) + .45(13%) + .25(15.7%) + [.30(1.2) + .45(.98) + .25(1.37)](*RM* – 14.2%)

+ .30εA + .45εB + .25εC

*RP* = 12.925% + 1.1435(*RM* – 14.2%) + .30εA + .45εB + .25εC

*c.* Using the market model, if the return on the market is 15 percent and the systematic risk is zero, the return for each individual stock is:

*R*A = 10.5% + 1.20(15% – 14.2%)

*R*A = 11.46%

*R*B = 13% + .98(15% – 14.2%)

*R*B = 13.78%

*R*C = 15.70% + 1.37(15% – 14.2%)

*R*C = 16.80%

To calculate the return on the portfolio, we can use the equation from part *b*, so:

*RP* = 12.925% + 1.1435(15% – 14.2%)

*RP* = 13.84%

Alternatively, to find the portfolio return, we can use the return of each asset and its portfolio weight, or:

*RP* = *X*1*R*1 + *X*2*R*2 + *X*3*R*3

*RP* = .30(11.46%) + .45(13.78%) + .25(16.80%)

*RP* = 13.84%

**7.** *a.* Since the five stocks have the same expected returns and the same betas, the portfolio also has the same expected return and beta. However, the unsystematic risks might be different, so the expected return of the portfolio is:

= 11% + .84*F*1 + 1.69*F*2 + (1/5)(ε1 + ε2 + ε3 + ε4 + ε5)

*b.* Consider the expected return equation of a portfolio of five assets that we calculated in part *a.* Since we now have a very large number of stocks in the portfolio, as:

N → ∞, → 0

But, the εjs are infinite, so:

(1/N)(ε1 + ε2 + ε3 + ε4 +…..+ εN) → 0

Thus:

= 11% + .84*F*1 + 1.69*F*2

*Challenge*

**8.** To determine which investment an investor would prefer, you must compute the variance of portfolios created by many stocks from either market. Because you know that diversification is good, it is reasonable to assume that once an investor has chosen the market in which she will invest, she will buy many stocks in that market.

Known:

E*F* = 0 and σ = .10

Eε = 0 and Sεi = .20 for all i

If we assume the stocks in the portfolio are equally-weighted, the weight of each stock is , that is:

Xi =  for all i

If a portfolio is composed of N stocks each forming 1/N proportion of the portfolio, the return on the portfolio is 1/N times the sum of the returns on the N stocks. To find the variance of the respective portfolios in the two markets, we need to use the definition of variance from statistics:

Var(x) = E[x – E(x)]2

In our case:

Var(*RP*) = E[*RP* – E(*RP*)]2

Note however, to use this, first we must find *RP* and E(*RP*). So, using the assumption about equal weights and then substituting in the known equation for *R*i:

*RP* = 

*RP* = (.10 + β*F* + ε*i*)

*RP* = .10 + β*F* + 

Also, recall from statistics a property of expected value, that is:

If: 

where *a* is a constant, and , , and are random variables, then:



and

E(*a*) = *a*

Now use the above to find E(*R*P):

E(*RP*) = E

E(*RP*) = .10 + βE(*F*) + 

E(*RP*) = .10 + β(0) + 

E(*RP*) = .10

Next, substitute both of these results into the original equation for variance:

Var(*RP*) = E[*R*P – E(*R*P)]2

Var(*RP*) = E

Var(*RP*) = E

Var(*RP*) = E

Var(*RP*) = 

Finally, since we can have as many stocks in each market as we want, in the limit, as N → ∞,

 → 0, so we get:

Var(*RP*) = β2σ2 + Cov(ε*i*,ε*j*)

and, since:

Cov(ε*i*,ε*j*) = σ*i*σ*j*ρ(ε*i*,ε*j*)

and the problem states that σ1 = σ2 = .10, so:

Var(*RP*) = β2σ2 + σ1σ2ρ(ε*i*,ε*j*)

Var(*RP*) = β2(.01) + .04ρ(ε*i*,ε*j*)

So now, to summarize what we have so far:

*R*1i = .10 + 1.5*F* + ε1i

*R*2i = .10 + .5*F* + ε2i

E(*R*1P) = E(*R*2P) = .10

Var(*R*1P) = .0225 + .04ρ(ε1*i*,ε1*j*)

Var(*R*2P) = .0025 + .04ρ(ε2*i*,ε2*j*)

Finally we can begin answering parts *a*, *b*, and *c* for various values of the correlations:

*a.* Substitute ρ(ε1*i*,ε1*j*) = ρ(ε2*i*,ε2*j*) = 0 into the respective variance formulas:

Var(*R*1P) = .0225

Var(*R*2P) = .0025

Since Var(*R*1P) > Var(*R*2P), and expected returns are equal, a risk-averse investor will prefer to invest in the second market.

*b.* If we assume ρ(ε1*i*,ε1*j*) = .9, and ρ(ε2*i*,ε2*j*) = 0, the variance of each portfolio is:

Var(*R*1P) = .0225 + .04ρ(ε1*i*,ε1*j*)

Var(*R*1P) = .0225 + .04(.9)

Var(*R*1P) = .0585

Var(*R*2P) = .0025 + .04ρ(ε2*i*,ε2*j*)

Var(*R*2P) = .0025 + .04(0)

Var(*R*2P) = .0025

Since Var(*R*1P) > Var(*R*2P), and expected returns are equal, a risk-averse investor will prefer to invest in the second market.

*c.* If we assume ρ(ε1*i*,ε1*j*) = 0, and ρ(ε2*i*,ε2*j*) = .5, the variance of each portfolio is:

Var(*R*1P) = .0225 + .04ρ(ε1*i*,ε1*j*)

Var(*R*1P) = .0225 + .04(0)

Var(*R*1P) = .0225

Var(*R*2P) = .0025 + .04ρ(ε2*i*,ε2*j*)

Var(*R*2P) = .0025 + .04(.5)

Var(*R*2P) = .0225

Since Var(*R*1P) = Var(*R*2P), and expected returns are equal, a risk-averse investor will be indifferent between the two markets.

*d.* Since the expected returns are equal, indifference implies that the variances of the portfolios in the two markets are also equal. So, set the variance equations equal, and solve for the correlation of one market in terms of the other:

Var(*R*1P) = Var(*R*2P)

.0225 + .04ρ(ε1*i*,ε1*j*) = .0025 + .04ρ(ε2*i*,ε2*j*)

ρ(ε2*i*,ε2*j*) = ρ(ε1*i*,ε1*j*) + .5

Therefore, for any set of correlations that have this relationship (as found in part *c*), a risk adverse investor will be indifferent between the two markets.

**9.** *a.* In order to find standard deviation, σ, you must first find the variance, since σ = . Recall from statistics a property of variance:

If: 

where *a* is a constant, and , , and are random variables, then:



and:

Var(*a*) = 0

The problem states that return-generation can be described by:

*Ri,t* = αi + β*i*(*RM*) + ε*i,t*

Realize that *Ri,t*, *R*M, and ε*i,t* are random variables, and α*i* and β*i* are constants. Then, applying the above properties to this model, we get:

Var(*Ri*) = Var(*RM*) + Var(ε*i*)

and now we can find the standard deviation for each asset:

= .702(.0121) + .01 = .015929

=  = .1262, or 12.62%

= 1.22(.0121) + .0144 = .031824

=  = .1784, or 17.84%

= 1.52(.0121) + .0225 = .049725

=  = .2230, or 22.30%

*b.* From the above formula for variance, note that as N → ∞,  → 0, so you get:

Var(*Ri*) = Var(*RM*)

So, the variances for the assets are:

= .72(.0121) = .005929

 = 1.22(.0121) = .017424

 = 1.52(.0121) = .027225

*c.* We can use the model:

= *RF* + βi(– *RF*)

which is the CAPM (or APT Model when there is one factor and that factor is the Market). So, the expected return of each asset is:

= 3.3% + .70(10.6% – 3.3%) = 8.41%

= 3.3% + 1.2(10.6% – 3.3%) = 12.06%

= 3.3% + 1.5(10.6% – 3.3%) = 14.25%

We can compare these results for expected asset returns as per CAPM or APT with the expected returns given in the table. This shows that Assets A and B are accurately priced, but Asset C is overpriced (the model shows the return should be higher). Thus, rational investors will not hold Asset C.

*d.* If short selling is allowed, rational investors will sell short Asset C, causing the price of Asset C to decrease until no arbitrage opportunity exists. In other words, the price of Asset C should decrease until the return becomes 14.25 percent.

**10.** *a.* Let:

*X*1 = the proportion of Security 1 in the portfolio, and

*X*2 = the proportion of Security 2 in the portfolio

and note that since the weights must sum to 1.0:

*X*1 = 1 – *X*2

Recall from Chapter 11 that the beta for a portfolio (or in this case the beta for a factor) is the weighted average of the security betas, so:

β*P*1 = *X*1β11 + *X*2β21

β*P*1 = *X*1β11 + (1 – *X*1)β21

Now, apply the condition given in the hint that the return of the portfolio does not depend on F1. This means that the portfolio beta for that factor will be 0, so:

β*P*1 = 0 = *X*1β11 + (1 – *X*1)β21

β*P*1 = 0 = *X*1(1.0) + (1 – *X*1)(.5)

and solving for *X*1 and *X*2:

*X*1 = – 1

*X*2 = 2

Thus, sell short Security 1 and buy Security 2.

To find the expected return on that portfolio, use:

*RP* = *X*1*R*1 + *X*2*R*2

so applying the above:

E(*RP*) = –1(20%) + 2(20%)

E(*RP*) = 20%

β*P*1 = –1(1) + 2(.5)

β*P*1 = 0

*b.* Following the same logic as in part *a*, we have

β*P*2 = 0 = *X*3β31 + (1 – *X*3)β41

β*P*2 = 0 = *X*3(1) + (1 – *X*3)(1.5)

and

*X*3 = 3

*X*4 = –2

Thus, sell short Security 4 and buy Security 3. Then,

E(*RP*2) = 3(10%) + (–2)(10%)

E(*RP*2) = 10%

β*P*2 = 3(.5) – 2(.75)

β*P*2 = 0

Note that since both βP1 and βP2 are 0, this is a risk-free portfolio!

*c.* The portfolio in part *b* provides a risk-free return of 10 percent, which is higher than the 5 percent return provided by the risk-free security. To take advantage of this opportunity, borrow at the risk-free rate of 5 percent and invest the funds in a portfolio built by selling short Security 4 and buying Security 3 with weights (3,–2) as in part *b*.

*d.* First assume that the risk-free security will not change. The price of Security 4 (that everyone is trying to sell short) will decrease, and the price of Security 3 (that everyone is trying to buy) will increase. Hence the return of Security 4 will increase and the return of Security 3 will decrease.

The alternative is that the prices of Securities 3 and 4 will remain the same, and the price of the risk-free security drops until its return is 10 percent.

Finally, a combined movement of all security prices is also possible. The prices of Security 4 and the risk-free security will decrease and the price of Security 3 will increase until the opportunity disappears.

