**Explanations of Quiz Solutions for Chapter 4**

1. The amount of money you are borrowing is the present value of the mortgage. Since the payments are being amortized over 30 years with monthly payments, this is an annuity with a monthly interest rate and 360 months. If the APR is 7.5%, the monthly rate must be .075/12. Using the PMT function in Excel, we find the monthly payments.

The payoff on any loan is the present value of the future payments. We just found that the monthly payments were $1,398.43. There are 20 years (240 months) of monthly payments remaining. The monthly interest rate is still .075/12. We can use the PV function in Excel to find the present value of this ordinary annuity. It is an ordinary annuity because the next payment is due at the end of the month (not immediately due).

Since the next payment is immediately due, this is an annuity due. There are 240 payments still to be made (just like the problem above), but instead of the next payment being due a month from now, it is due right away – so naturally, the payoff will be greater by one month’s interest. Use the PV function in Excel to find the solution with a “1” as type since it is an annuity due.

1. If you have monthly payments, you must solve for the monthly interest rate. Use the RATE function in Excel with the purchase price as the present value and the number of monthly payments as the number of periods. If the first monthly payment is today, and the last one is five years from today, that is 61 monthly payments. Since the first payment is today and the present value is its value today, this is an annuity due. This means you will need to use “1” as the argument for “type”. Be sure to make enter either the 10,000 or the 200 as a negative (cash outflow) in Excel. Once Excel solves for the monthly rate, we can multiply that by 12 months/yr to find the APR. To find the EAR, add one to the monthly rate, raise it to the 12th power and then subtract one.
2. You are solving for both the future value of an annuity and the future value of the money you have already saved. There is only one interest rate, so both values must use the same rate. The interest rate is 7% APR, but with monthly compounding, that makes it a monthly rate of 7%/12. For the annuity, there will be 300 payments into the retirement plan. The next payment will be made a month from now, so this is an ordinary annuity. For the existing $50,000, the money is being compounded over 300 months. In Excel, you can calculate the future value of both parts at the same time using the FV function with 50,000 as the PV and 500 as the PMT.
3. I suggest drawing a timeline to more easily see where the cash flows occur. For each child, the tuition payments are a delayed annuity. For the oldest child, the payments start at time 15 and for the youngest child, they start at time 17. Since the present value of an annuity formula gives us the value as of one period before the first cash flow, finding the present value of four $30,000 payments beginning at time 15 will give us the value at time14 for the oldest child. With the payments beginning at time 17 for the youngest child, the formula gives us the value at time 16. In each case, we use the PV function in Excel with NPER as 4 and PMT as 30,000. Of course, we only need to do this once since it will give us the same value for each child. Once we have the present value of the annuities at times 14 and 16, we discount each to time zero. This means using the PV function again, and referring to the cell where we got $99,363.81 as our FV. For the oldest child, NPER is 14 because the tuition payments, which started at time15, have already been discounted to time 14. For the youngest child, NPER is 16. We now have the present value of all the tuition payments at time zero. Since they are in the same time period, we can add them together to get $62,832.85. We need the present value of the contributions to the college fund to equal the present value of the tuition payments. Since the first contribution will be at the end of the year, and there will be 15 contributions, this is an ordinary annuity with 15 payments. We use the PMT function in Excel to solve for the annual contributions.
4. We need to use the NPER function in Excel to find the number of payments needed to pay off a $100,000 mortgage when you are making payments of $1,000 per month. This is a present value of an annuity problem. Be sure to enter either the PV or the PMT amount as a negative value. Since these are monthly payments, we need to use the monthly interest rate (6%/12). Excel’s solution to the math problem is 138.98. This means that we will make 138 monthly payments of $1,000 and still owe $970.93. So while the 139th payment will not need to be a full $1,000, you will need to make some payments for 139 months which means you are paying off your mortgage 41 months sooner than the 180 months (15 years) it would have taken you with the scheduled payments.
5. We start by finding the present value of the lease payments. Since the first payment is due immediately, this is an annuity due. We can solve this using the PV function in Excel. Note that if you are making your first lease payment today and the last one exactly three years from today, you are making 37 payments. The present value of the lease payments is $14,810.31. Next we need to find the present value of the cash flows if you buy the car. Of course, the present value of buying the car will be the purchase price of $35,000. The difference between buying and leasing is that when you buy the car, you still own it after the three years. The true cost of owning the car means discounting the future sale price and subtracting that from the purchase price. There is only one interest rate and it is a monthly rate of 8%/12. We use this rate to discount the $23,000 sales price which takes place 36 months from now. This means that the net cost of buying the car is (in present value dollars) $16,893.14.

We now want to find out what resale price at time 36 would cause the present value of buying the car equal to the present value of leasing the car (already found to be $14,810.31) and thus make us indifferent between buying and leasing. Since the net cost of buying the car is $35,000 minus the present value of the resale price, we need to solve for the resale price which will make the net cost $14,810.31. We can solve this using basic algebra or, if you want, using SOLVER in Excel.

1. An APR of 6 percent means that when we have annual compounding, the “true” interest rate is 6%, when we have quarterly compounding, it is 6%/4, and when we have daily compounding it is 6%/365. For parts A-C, you can solve for the future value by using the FV function in Excel. In each case, $5,000 is the PV and the RATE is 6% divided by either 1, 4 or 365. The NPER is the number of compounding periods over 10 years. For continuous compounding, we use the formula FV = PV ert. Where r is the stated rate per period (6% per year) and t is the number of periods (10). In Excel, we use the EXP function to find e raised to some power, with the exponent placed in the parentheses. Once we have that, we multiply it by $5,000 to find the future value. To find the EAR with quarterly and daily compounding, just take the quarterly or daily rate (6%/4) or 6%/365), raise it to the t power (4 or 365), and subtract one. For continuous compounding, it is eAPR -1. With annual compounding, the APR and the EAR are one and the same.
2. Since you make monthly payments on the car loan, it is safe to assume that there is monthly compounding. That means that the monthly interest rate is 6.5%/12. To find the monthly payment, use the present value of an annuity formula with the present value being the amount of the loan, the rate being the monthly interest rate, and the number of periods being the number of months (12 x 4 = 48) you are making the payments. Solve for “C”, which is the monthly payment. At the end of the first month, you have borrowed $35,000 for one month, so you owe one month’s worth of interest on $35,000. Since the monthly interest rate is 6.5%/12, you multiply that by how much you borrowed to find out how much interest you owe. Since we have already determined that you are paying $830.02 each month, and you owe $189.58 in interest at the end of the first month, the remainder of your payment must be principal. So just subtract the interest payment from the total payment to find the principal payment for that month.
3. Option 1 is an annuity due since you receive the first payment immediately. To find the present value of an annuity due, use the present value of an annuity formula and multiply it by (1+r). In excel, you just enter “1” in the “type” box in the function wizard fo present value of an annuity. Of course, you must also enter $10,000 as the payment, 6% as the rate, nad 30 as the number of preriods. Option 2 is a delayed annuity since you must wait 10 years to get the first payment. Finding the present value of a delayed annuity requires two steps. The first step involves finding the present value of the annuity as of one time period before the first payment comes. Since the first payment comes at time 10, this means you will find the present value at time 9. Just enter everything into the function wizard as you normally would (number of periods equals 20 since you are getting 20 payments of $20,000 each), to get $229,398.42 as the present value at time 9. Note we discounted the cash flows in times 10-19 back to time 9. Now we have to bring the $229,398.42 at time 9 back to time zero. It is now a single lump sum at time 9, so we just use the present value formula with the rate being 6% and the number of periods being 9. Be sure to reference the cell where you got your answer of $229,398.42 as the future value. Do not re-type it in or your final answer may be slightly off. Clearly option 1 has the higher present value, but the problem doesn’t ask you to choose one option or the other.
4. Since you expect to drive your car 400 miles per week for the next eight years, it is logical to start by determining how much money (in gasoline costs) the hybrid will save you each week. At $3.50 per gallon and 25 miles/gallon, the regular car costs 14 cents per mile, or $56 per week. The hybrid costs 10 cents per mile, or $40 per week. This means you save $16 per week if you buy the hybrid. But does that justify the higher price tag of the hybrid? To find out, we need to calculate the present value of that $16 per week savings over the lifetime of the car (8 years, or 416 weeks). Since this is a weekly savings, we must use the weekly interest rate (7%/52) and the number of weeks (416). Using the PV function in excel, we find that the hybrid car saves us $5,093.93 in present value terms. Since the hybrid only costs $4,000 more than the regular car, but saves us more than $5,000 (comparing present value with present value), it is more economical.