**FINE 7110**

**Fall 2024**

Duration and Bootstrapping Assignment - **Solutions**

* 1. What is the price of a bootstrapped 2.2% (coupon rate) 10-year Treasury Note?

**82.77**

* 1. What is the YTM of the Treasury Note in question a?

**4.34%**

* 1. Price a 10-year 2.2% corporate bond so that it has a 50-basis point credit spread over the Treasury note you bootstrapped.

**79.253**

* 1. Using the zero-coupon rates (the semiannual z values) that were found in the bootstrapping spreadsheet, find what the YTM would be for a previously-issued Treasury bond that matures in exactly two years and is currently selling at par value.

**4.26%**

* 1. Repeat a-d above for a Treasury Note and a corporate bond with a 3.5% coupon rate.

1. **93.25**
2. **4.34%**
3. **89.48**
4. **4.26% (Same)**
   1. Look at the Zero Coupon Yield Curve that the bootstrapping spreadsheet made. How would you describe the shape of this yield curve?

**Mostly downward sloping for next 7 year. Upward sloping years 7-10**

* 1. Based on the Unbiased Expectations Theory of the term structure, what does this yield curve tell us about investors’ expectations over the next ten years?

**Investors expect short-term rates to go down over the next seven years – then to increase for the following three years.**

1. You manage a pension plan with $200 million in assets that have a duration of 10 years and $100 million in liabilities with a duration of 4 years.
   1. Calculate your duration gap

**DURGap = 10 – (100/200 x 4) = 8**

* 1. Are you hurt if interest rates go up, or if they go down?

**If interest rates go up, the value of both your assets and your liabilities will go down, but the value of your assets will go down by more, so you are hurt if interest rates go up.**

* 1. Suppose you want to execute a macro hedge using T-bond futures contracts traded on the CBOT. Do you want to go short or long?

**The futures contract will go down in value if interest rates go up. This will offset what happens to your pension plan. So go short.**

* 1. Suppose that the cheapest to deliver bond has a duration of 25.3 years. How many contracts will you short/long if you want to hedge your position as completely as possible?

**VF x DURF = -(VA x DURGap)**

**VF x 25.3 = -($200 million x 8) → VF = -$63,241,106**

**At $100,000 per futures contract, go short 632 contracts**

**Or sell 632 contracts**

1. You have been hired as a consultant by Steady-State Life Insurance Company. Steady-State specializes in the sale of Guaranteed Investment Contracts (GICs) which guarantee clients a specific dollar amount on a specific date in return for either a single up-front premium payment or a series of monthly payments.

Steady-State has just sold a GIC which guarantees a payout of $1.52 million on January 1, 2034. Steady-State needs to purchase a high quality bond that will assure them of having the $1.52 million on 1/1/34. They are willing to spend up to $950,000 on a bond to fund this GIC, and plan to buy it today, January 1, 2024.

Currently, the yield curve is flat so that the YTM of every bond with the credit-quality Steady-State is considering is currently 5.00%. You expect the yield curve to remain flat for at least the next 15 years, even though interest rates could go up or go down during that time.

Your job is to recommend a bond to purchase which will assure Steady-State of having the necessary $1.52 million in 10 years no matter how high or low interest rates might go between now and then. This is the only objective – make sure that the investment will grow to at least $1.52 million on January 1, 2034.

Steady-State has asked you to consider the following three corporate bonds that meet their standards for credit quality and liquidity. Each bond makes semi-annual coupon payments.

**Bond #1**

Face Value: $1 million

Coupon Rate: 4.25%

Maturity: January 1, 2034

**Bond #2**

Face Value: $1 million

Coupon Rate: 4.25%

Maturity: January 1, 2037

**Bond #3**

Face Value: $1 million

Coupon Rate: 4.375%

Maturity: January 1, 2039

Please answer the following:

1. Find the current price for each bond and ensure that it is less than $950,000

**Bond #1 - $941,540.64**

**Bond #2 - $928,935.21**

**Bond #3 - $934,592.84**

1. Calculate the duration of each bond

**Bond #1 – 8.1492**

**Bond #2 – 9.9916**

**Bond #3 – 10.999**

1. For each bond, determine how much money you will have on January 1, 2034 under each of the following three scenarios:
2. Interest rates stay at 5% (2.5% semiannually) for at least the next 15 years

**Bond #1 - $1,542,823.97**

**Bond #2 - $1,522,168.50**

**Bond #3 - $1,531,439.19**

1. Tomorrow, after you have purchased the bond, interest rates immediately go up to 6% and stay there for the next 15 years

**Bond #1 - $1,570,995.46**

**Bond #2 - $1,523,595.03**

**Bond #3 - $1,518,481.54 which is less than $1.52 million**

1. Tomorrow, after you have purchased the bond, interest rates immediately go down to 4% and stay there for the next 15 years

**Bond #1 - $1,516,319.11 which is less than $1.52 million**

**Bond #2 - $1,523,320.90**

**Bond #3 - $1,548,347.31**

Make note of the fact that Bond #1 matures when you need the $1.52 million, but Bond #2 and Bond #3 must be sold prior to maturity (sold on 1/1/34). Of course, they will sell for the present value of their remaining cash flows at that time.

Also note that all coupon payments will be reinvested at the prevailing interest rate when they are received.

So, for each bond, on 1/1/34, you will have the future value of the reinvested coupon payments plus either the face value of the bond (#1) or the sale price of the bond (#2 and #3).

1. Present a recommendation of which of the three bonds Steady-State should buy, and why.

**Buy Bond #2 because it will give you at least $1.52 million no matter what interest rates do**

1. Explain how an understanding of duration allows you to pick the correct bond without doing any of these calculations.

**Matching the duration of your assets with the duration of your liabilities ensures that you are hedged. The liability and Bond #2 both have a duration of (about) 10.**

1. The correct bond will provide Steady-State with **more** than the needed $1.52 million if interest rates go up or if they go down. Explain what property of this (correct) bond causes this and calculate its value for both the bond and the GIC to prove it.

**Bond #2 is more convex than the liability, so whether interest rates go up or down, it will have more than $1.52 million.**

**Convexity of Liability – 99.94**

**Convexity of Bond #2 – 116.72**